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Connectivity of cognitive radio ad hoc networks with directional antennas

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Abstract In cognitive radio ad hoc networks, omni-directional antennas are typically used at both primary users (PUs) and secondary users (SUs), which can cause high interference. We name such cognitive radio ad hoc networks with omni-directional antennas as OMN-CRAHNs. Different from omni-directional antennas, directional antennas can concentrate the transmission on desired directions and can consequently reduce interference in undesired directions. In this paper, we investigate both the local connectivity and the overall connectivity of cognitive radio ad hoc networks with directional antennas (DIR-CRAHNs), in which both PUs and SUs are equipped with directional antennas. In particular, we establish a theoretical framework to analyze both the probability of node isolation and the probability of connectivity of DIR-CRAHNs and OMN-CRAHNs. Our analytical results show that DIR-CRAHNs can have higher connectivity than OMN-CRAHNs.

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1 Introduction

There are growing demands for higher data rate due to the emergence of bandwidth-ravenous applications (e.g., 4 k video streaming) while the current wireless networks cannot offer a solution to the drastic growth of high bandwidth demands mainly owing to the *underutilized radio spectrum* [28, 44]. One reason is the *fixed-spectrum-allocation-scheme*, in which radio spectrum is allocated only for legitimate users. Once the spectrum is allocated for the dedicated users, no unsubscribed users can use the spectrum. However, this fixed-spectrum allocation-scheme is not efficient because a large portion of the licensed spectrum has been underutilized especially when legitimate users are idle. Recently, *cognitive radio* technology has been proposed to improve the spectrum reusage by allowing unlicensed users to opportunistically use the spectrum [1].

The network connectivity, a fundamental property in wireless ad hoc networks (WAHNs), has received extensive attentions [5, 6]. The network connectivity measures the probability that a sender can successfully communicate with its receiver. Different from a homogeneous WAHN, a cognitive radio wireless ad hoc network (CRAHN) consists of two types of users: (i) Primary users using the licensed spectrum and (ii) Cognitive (Secondary) users using the licensed spectrum *opportunistically* (e.g., in underlay CRAHNs, second users can only use the spectrum when the interference is no greater than a threshold). For simplicity, we call the network formed by primary users as PNets and call the network formed by Secondary users as

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SNets. The network connectivity of SNets is usually more difficult to be achieved than that of PNets due to the less available spectrum for cognitive users than that for primary users [3, 26]. In such CRAHNs, both PNets and SNets consist of users equipped with only omni-directional antennas, which cause high interference. We name such CRAHNs with omni-directional antennas as OMN-CRAHNs.

Recent works such as [8, 39, 51] show that applying directional antennas instead of omni-directional antennas to wireless networks can greatly improve the network performance (e.g., capacity and connectivity). Compared with omni-directional antennas, directional antennas can concentrate the radio signals on desired directions. In other undesired directions, there are no radio signals or only weakened signals. Thus, using directional antennas in wireless networks can potentially reduce the interference, improve the spectrum reuse and consequently improve the network performance. One interesting question is whether using directional antennas in CRAHNs can also improve the network connectivity.

In this paper, we consider underlay cognitive radio ad hoc networks with directional antennas named as *DIR-CRAHNs* that have the following characteristics.

- Each primary user (node) is mounted with a *directional* antenna, which can legitimately use the licensed spectrum.
- Each secondary user (node) is mounted with a *directional* antenna, which can use the licensed spectrum only when the interference of the secondary user is lower than a threshold).
- The primary nodes form an ad hoc network (i.e. PNet) and the secondary nodes form an ad hoc network (i.e., SNet).

There are few studies on DIR-CRAHNs. Though Wei et al. [42] investigated the asymptotic throughput of CRAHNs with directional transmissions, they derived the throughput based on the asymptotically connected CRAHNs with simple extension from CRAHNs with omni-directional antennas. Their study does not offer a general analytical framework on network connectivity for DIR-CRAHNs (such as the local connectivity and the overall multi-hop connectivity). *To the best of our knowledge, there is no study on establishing the analytical model on the network connectivity of DIR-CRAHNs.*

In this paper, we investigate the network connectivity of DIR-CRAHNs, which is believed to be one of first studies in this new area. We summarize our contributions as follows.

- We propose a theoretical framework to investigate both the local and the overall connectivity of the DIR-CRAHNs.
- We consider both DIR-CRAHNs and OMN-CRAHNs in the same theoretical framework and OMN-CRAHNs may become a special case of DIR-CRAHNs in some scenarios, implying that our analytic model is quite general.
- We also show that DIR-CRAHNs can have a better network connectivity than conventional OMN-CRAHNs due to the usage of directional antennas, which cause the less interference and improve the spectrum reuse.

The rest of this paper is organized as follows. Section 2 summarizes the related works. In Sect. 3, we present the models used in this paper. We first analyze the local connectivity of DIR-CRAHNs and OMN-CRAHNs in Sect. 4. We then derive the overall connectivity of DIR-CRAHNs and OMN-CRAHNs in Sect. 5. Finally, Sect. 6 concludes this paper.

2 Related works

The network connectivity is an important measure of the performance of wireless ad hoc networks. Essentially, the network connectivity is a necessity to ensure the network is connected so that each source node can successfully communicate with its destination node. Gupta and Kumar [17] investigated the asymptotic connectivity of wireless ad hoc networks (WAHNs) by deriving a sufficient and necessary condition to ensure that the network is connected. Bettstetter [5] proposed an analytical framework on network connectivity based on the probabilistic theory of random networks. Besides, the impacts of various random channel models on the network connectivity were considered in [29]. However, most of the above studies only consider WAHNs equipped with omni-directional antennas, which radiate radio signals in all directions. On one hand, an omni-directional antenna can lead to extra interference in other undesired directions. On the other hand, an omni-directional antenna also results in the short transmission range. Both of the two drawbacks of omni-directional antennas lead to the poor network connectivity of WAHNs. Recently, many studies such as [8, 9, 24, 47] show that using directional antennas in WAHNs can significantly improve the network performance. However, it is difficult for each node to obtain the location knowledge of other neighbors due to the directional beamforming [9]. To solve the directional neighbor discovery problem, the complicated schemes such as direction-of-arrival (DOA) estimation [38] and swiveling-beam scheme [32, 49] were proposed. It is shown in [6, 15, 39, 51] that using directional antennas in WAHNs can improve the network connectivity when the directional neighbor discovery problem is solved.

Compared with conventional WAHNs, CRAHNs can improve the spectrum reuse [50]. There are a number of studies on the performance improvement of SUs of CRAHNs [14, 25]. The network connectivity of CRAHNs has recently received extensive attentions. In particular, the key differences between CRAHNs and conventional WAHNs have been discussed in [1, 12]. Ren, Zhao and Swami are the pioneers to investigate the 1-connectivity of CRAHNs [35]. Moreover, k-connectivity of CRAHNs was investigated in [13]. Both the local connectivity and the overall connectivity of CRAHNs were analyzed in [26, 48].

Table 1 summarizes the aforementioned studies on WAHNs and CRAHNs from the different aspects of omni-directional and directional antennas. In particular, only [42] and this paper investigate the connectivity of CRAHNs with directional antennas. Although [42] analyzed the throughput on asymptotically connected CRAHNs with directional transmissions, it is just a direct extension from conventional OMN-CRAHNs [41]. There is no concrete analytical model proposed in [42] to investigate either the *local network connectivity* or the *overall connectivity* of DIR-CRAHNs. Therefore, it is the goal of this paper to investigate the network connectivity of CRAHNs with directional antennas.

In our previous paper [40], we conducted a preliminary study on the local connectivity of DIR-CRAHNs. However, this paper is significantly different from our previous work [40] in the following aspects: (1) we analyze both the local connectivity (in Sect. 4) and the overall connectivity (in Sect. 5) in this paper while our previous conference paper only investigates the local connectivity; (2) we consider a safe zone for SUs to ensure the more reliable transmissions of SUs in this paper (note that safe zone or guard zone has been proposed and used in previous studies such as [21, 22]); (3) we present more numerical results with consideration of both the local connectivity and the overall connectivity.

3 System models

Section 3.1 presents the network model. In our DIR-CRAHNs, both PUs and SUs are equipped with directional antennas, which will be introduced in Sect. 3.2. Section 3.3 then presents the channel model. Finally, we define link criterion for SUs in Sect. 3.4.

Table 1 Summary of connectivity on WAHNs and CRAHNs

	WAHNs	CRAHNs
Omni-directional antennas	[5, 29]	[13, 26, 35, 48]
Directional antennas	[6, 15, 39, 51]	[42], this paper

3.1 Network model

There are several cognitive radio spectrum sharing paradigms in CRAHNs: *underlay* paradigm, *overlay* paradigm and *interweave* paradigm [16]. In this paper, we only consider the underlay paradigm because it can best utilize the spectrum [16]. In CRAHNs, there are two kinds of users: Primary users (PUs) and Secondary users (SUs). In the under-laid paradigm, PUs use the licensed spectrum and SUs can only use the spectrum when the interference to active PUs is no greater than a threshold. For simplicity, we call the network formed by PUs as PNets and call the network formed by SUs as SNets. In this paper, we mainly concentrate on the network connectivity of SNets since the network connectivity of PNets is usually ensured due to the higher spectrum availability of PUs than that of SUs.

The performance of SUs is highly dependent upon the PU activity model [23, 37]. In this paper, we assume that PUs are distributed according to a two-dimensional Poisson point process with density λ_p. Without loss of generality, we also consider that all PUs use the same transmission power. Similar to the previous studies [11, 26, 27], we further assume that the traffic arrival rate of PUs follows a Poisson process with density λ_{arr}. Thus, the probability of q packets arriving at a PU in unit time is $\mathbb{P}_{PU}^{arr}(q) = \frac{\lambda_{arr}^q}{q!} e^{-\lambda_{arr}}$ [36]. It is worth mentioning that there are a number of recent studies on modeling the activity of PUs as summarized in [37] including ON/OFF model, Markov chain, queuing theory, etc. In this paper, we consider this simplified model because (i) it can be used to obtain the closed-form expression on the connectivity; (ii) our main concern lies in the impact of other research aspects such as the impact of spectrum availability on the connectivity. Investigating the impacts of different activity models of PUs on the connectivity of CRAHNs will be one of our future studies.

SNets consist of SUs that are distributed according to a two-dimensional Poisson point process with density λ_s. In this paper, we mainly concern with the impacts of PUs on the network connectivity of SNets. Besides, SUs are smart devices and they can avoid interference to PUs and other SUs (i.e., cognitive radio technology can achieve this goal). Therefore, we only consider the interference imposed from SUs to PUs. Moreover, we have a mechanism to protect the communications of PNets: when there is a PU that is active (either transmitting or receiving) within the transmission region of an SU, we prohibit this SU from transmission. Consequently, the path with inclusion of this SU will be disconnected. That is the reason why the network connectivity of SNets is usually more difficult to be ensured than that of PNets [3, 26].

In DIR-CRAHNs, both PUs and SUs are equipped with directional antennas only. Differently, in conventional OMN-CRAHNs, both PUs and SUs are equipped with omni-directional antennas only. Section 3.2 will present the antenna models for directional antennas and omni-directional antennas. Note that we do not consider the case that PUs are equipped with directional antennas and SUs are equipped with omni-directional antennas and the case that PUs are equipped with omni-directional antennas and SUs are equipped with directional antennas. Because our DIR-CRAHNs can explore benefits of directional antennas completely while the above two cases cannot.

3.2 Antenna model

An *antenna* is a device that radiates (or collects) radio signals into (or from) space. There are different types of antennas used in modern wireless communication systems. We often categorize antennas into two types: *Omni-directional* antennas and *Directional antennas*. Conventional wireless networks are typically equipped with omni-directional antennas, which radiate/collect radio signals into/from all directions equally. Different from an omni-directional antenna, a directional antenna can concentrate transmitting or receiving capability on some desired directions. As a result, the received/emitted signals in directional antenna can be significantly enhanced compared with omni-directional antennas.

We often use the *antenna gain* to model the transmitting or receiving capability of an antenna. The antenna gain $G(\theta, \phi)$ defined in 3-D spherical coordinate system can be expressed as the following equation [4].

$$G(\theta, \phi) = \eta \frac{U(\theta, \phi)}{U_o}, \tag{1}$$

where θ is the angle from z-axis, ϕ is the angle from the x in the xy-plane ($\phi \in (0, 2\pi)$), η is the efficiency factor, which is set to be 1 since all the antennas in this paper are assumed to be loss-less, $U(\theta, \phi)$ is the *radiation intensity*, which is defined as the power radiated from an antenna per unit solid angle, and U_o denotes radiation intensity of an omni-directional antenna. The graphical representation of the antenna gain of an antenna in all the directions in 3-D space is called the *radiation pattern* of this antenna. Figure 1(a) shows the radiation pattern of an omni-directional antenna (also named *isotropic* antenna). It is obvious that an omni-directional antenna has antenna gain $G_o = 1$ because $U(\theta, \phi) = U_o$ since an omni-directional antenna radiates the power uniformly in all directions.

One of the most commonly used realistic directional antennas is *Uniform Circular Array (UCA)* antenna [4]. A UCA antenna consists of M omni-directional antenna elements uniformly distributed on a circle with radius a. For this array configuration, the radiation intensity $U(\theta, \phi)$ is given by

$$U(\theta, \phi) \propto |E(\theta, \phi)|^2, \tag{2}$$

where $E(\theta, \phi)$ denotes the far-zone electric field strength at a given direction (θ, ϕ), which can be calculated by the following equation [4].

$$E(\theta, \phi) = \sum_{m=1}^{M} I_m \cdot e^{jka[\sin\theta\cos(\phi-\phi_m) - \sin\theta_0\cos(\phi_0-\phi_m)]}, \tag{3}$$

where j is the imaginary unit and $j^2 = -1$, $k = 2\pi/\lambda$ where λ is the wavelength of the propagating signal. $\theta_0 = \pi/2$ (i.e., we consider on the xy-plane) and $\phi_0 \in (0, 2\pi)$ is the azimuth angle of the desired main beam. $\phi_m = 2\pi m/M$ is the angular position of mth element on xy-plane and I_m is the amplitude excitation of the mth element, which is always set to be 1.

Finally, the antenna gain $G(\theta, \phi)$ of a UCA antenna is expressed as the follow equation,

$$G(\theta, \phi) = \frac{|E(\theta, \phi)|^2}{\frac{1}{4\pi} \cdot \int_0^{2\pi} \int_0^{\pi} |E(\theta, \phi)|^2 \cdot \sin\theta d\theta d\phi}. \tag{4}$$

However, it is complicated to compute the antenna gain of a realistic antenna in each direction. Besides, realistic antenna model can not be used to solve the problem of deriving the optimal bounds on the network connectivity [19]. Thus, several simplified directional antenna models have been proposed. In particular, an approximated antenna model has been proposed in [31] and been widely used in [19, 24]. This model is named as *Sector* antenna model. As shown in Fig. 1(b), Sector model only consists of one main beam with *beamwidth* θ_m (equal to the HPBW of a realistic antenna) and all the side/back lobes are ignored. We next derive the antenna gain of Sector model.

From Eq. (1), we then have the antenna gain of Sector model as follows,

$$G(\theta, \phi) = \frac{U(\theta, \phi)}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\frac{\theta_m}{2}} U(\theta, \phi) \sin\theta d\theta d\phi} = \frac{2}{1 - \cos\frac{\theta_m}{2}}. \tag{5}$$

As shown in Eq. (5), $G(\theta, \phi)$ is a function of the beamwidth θ_m. For simplicity, we denote the antenna gain of Sector antenna model by G_d.

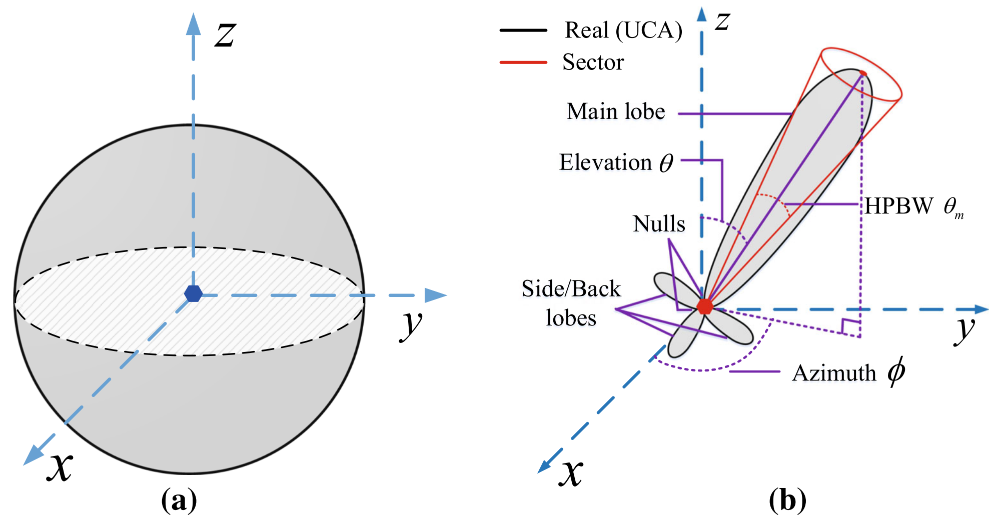

Fig. 1 Antenna models. **a** Omni-directional antenna model, **b** realistic versus sector model

3.3 Channel model

We assume that a node transmits with power P_t. Then, the receiving power at the receiver denoted by P_r can be calculated by

$$P_r = \frac{P_t \cdot G_t \cdot G_r}{10^{\omega/10} \cdot l^{\alpha}}, \tag{6}$$

where l is the distance between the transmitter and the receiver, G_t and G_r are the antenna gains of a transmitter and a receiver, respectively. In outdoor environment, the *channel gain* between a transmitter and a receiver is mainly determined by the path loss effect and shadow fading effect. In particular, the path loss effect is characterized by the path loss exponent α (usually $2 \le \alpha \le 6$ [34]). The shadow fading effect is modeled as log-normal random variable ω with zero mean and standard deviation σ (dB).

In practice, we usually compute the power attenuation [6] between two nodes instead of computing the received power P_r. We then introduce the power attenuation Δ defined as follows,

$$\Delta = \frac{P_t}{P_r} = \frac{10^{\omega/10} \cdot l^{\alpha}}{G_t \cdot G_r}. \tag{7}$$

In our DIR-CRAHNs, we assume that PUs and SUs have the different antenna settings. In particular, each PU is equipped with an antenna with beamwidth θ_p and each SU is equipped with an antenna with beamwidth θ_s, where θ_p is not necessarily equal to θ_s. We also assume that the direction of each antenna follows the uniform distribution in $[0,2\pi]$.

3.4 Link criterion

Link condition (criterion) is a fundamental measure of wireless ad hoc networks (WAHNs) and it may concern whether two nodes can establish a link with each other. It is more challenging to analyze the link condition in CRAHNs than that in conventional WAHNs. In conventional WAHNs, two nodes can establish a link between them if they fall into the transmission region of each other. However, in CRAHNs, the link criterion of two SUs depends on not only the transmission region but also the spectrum availability. In particular, the spectrum availability of SUs may change dynamically due to the activities of PUs. For example, as shown in Fig. 2, there shall exist two paths[1] from the source node SU_1 to the destination node SU_7 if SU_4 is not interfered by PU_1. However, the interference from PU_1 to SU_4 causes the abandonment of the transmission of SU_4. As a result, there is only one path available from SU_1 to SU_7 (i.e., from SU_1 through SU_3, SU_5 to SU_7

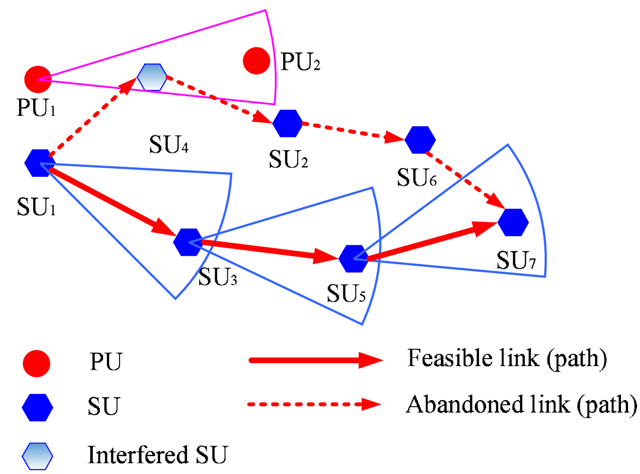

Fig. 2 Connectivity of DIR-CRAHNs

[1] Note that each SU in our DIR-CRAHNs is equipped with a single directional antenna, which can arbitrarily switch its antenna direction forward or backward. It can be easily achieved by *Switched Beam* antenna or *Adaptive Array* antenna [32].

as indicated as the solid arrow in Fig. 2). Intuitively, SNets of CRAHNs may have poorer connectivity than WAHNs due to the existence of PUs. We next analyze the link condition of CRAHNs. Specifically, we define the link condition of DIR-CRAHNs and the link condition of OMN-CRAHNs, respectively.

In this paper, we consider the bidirectional link since it can guarantee the deliver of acknowledgement in wireless networks (e.g., ACK has been used in Wireless Local Area Networks). The bidirectional link means that node SU_i and SU_j can establish a link if and only if SU_i and SU_j can successfully transmit to each other. Then, we define the link condition of DIR-CRAHNs and the link condition of OMN-CRAHNs as follows, respectively.

Definition 1 *Link Condition of DIR-CRAHNs* In DIR-CRAHNs, two nodes SU_i and SU_j are connected if and only if both the following conditions are satisfied.

1. The Euclidean distance between SU_i and SU_j is less than r_d, i.e., $\|\mathrm{SU}_i - \mathrm{SU}_j\| < r_d$, where SU_i and SU_j denote the positions of two SUs in a 2-D plane and r_d is the transmission range;
2. Both SU_i and SU_j have the available channel, which means that there are no active PUs in SUs' transmission region or the PUs shall not point their antennas to SUs if they fall into SUs' transmission region.

Note that in DIR-CRAHNs, we assume that the main directions of each pair of transmitter and receiver are pointed to each other to ensure the best link quality by using beam-locking mechanisms [38]. Besides, each SU in our DIR-CRAHNs is equipped with a single directional antenna, which can arbitrarily switch its antenna direction forward or backward. It can be easily achieved by *Switched Beam* antenna [20] or *Adaptive Array* antenna [32].

We then give the link condition of OMN-CRAHNs as follows.

Definition 2 *Link Condition of OMN-CRAHNs* In OMN-CRAHNs, two nodes SU_i and SU_j can establish a link if and only if both the following conditions are satisfied.

1. The Euclidean distance between SU_i and SU_j is less than r_o, i.e., $\|\mathrm{SU_i} - \mathrm{SU_j}\| < r_o$, where r_o is the transmission range;
2. Both SU_i and SU_j have the available channel, which means that there are no active PUs in SUs' transmission region.

Note that we only consider the link condition of two SUs without consideration of the inference caused by other SUs or PUs. This model is also called as the *pairwise* model, which has widely used in [5, 7, 51]. One of the reason of ignoring the interference of other users is mainly due to the higher path loss [7]. It is true that we can consider the interference from other SUs and PUs by using approaches such as stochastic geometry [2]. However, it will be challenging to derive the closed-form expression of the connectivity in CRAHNs when the interference is caused by different tiers of networks as implied in [10]. This will be left as one of our future studies.

The transmission range r_o can be obtained by Eq. (7). In particular, we let the power attenuation fixed at a given value Δ_t (a threshold) and $G_t = G_r = 1$, and then substitute them in Eq. (7). Then we have

$$r_o = \left(\frac{\Delta_t}{10^{\omega/10}}\right)^{\frac{1}{\alpha}}. \tag{8}$$

The transmission range r_d of DIR-CRAHNs can be calculated in a way similar to OMN-CRAHNs. Specifically, We also let Eq. (7) be equal to the given threshold Δ_t and $G_t = G_r = G_d$. Then we have

$$r_d = \left(\frac{\Delta_t \cdot G_d^2}{10^{\omega/10}}\right)^{\frac{1}{\alpha}}. \tag{9}$$

We assume that both OMN-CRAHNs and DIR-CRAHNs have the same network topologies and the same environment settings (e.g., the identical shadowing effect factor ω and σ). We then obtain the relationship between r_d and r_o after combining Eqs. (8) and (11), which is given by

$$r_d = r_o G_d^{\frac{2}{\alpha}}. \tag{10}$$

In our DIR-CRAHNs, we assume PUs and SUs can have different directional antenna settings. In particular, each SU is equipped with an antenna with beamwidth θ_s and each PU is equipped with an antenna with beamwidth θ_p. Then, the transmission range r_d can be expressed as

$$r_d = r_o \left(\frac{2}{1 - \cos\frac{\theta_s}{2}}\right)^{\frac{2}{\alpha}}. \tag{11}$$

It is implied from Eq. (11) that DIR-CRAHNs can extend the transmission range of SUs by $\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{2}{\alpha}}$ compared with OMN-CRAHNs.

4 Local network connectivity

In this section, we analyze the local connectivity of both DIR-CRAHNs and OMN-CRAHNs. We first derive the SU successful transmission probability in Sect. 4.1, which will then be used to derive the probability of node isolation in Sect. 4.2. We next present the analytical results in Sect. 4.3.

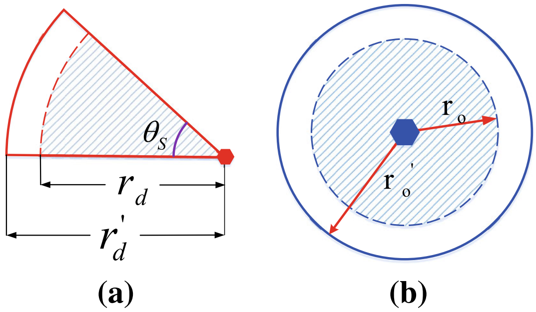

Fig. 3 Safe zone and transmission region. **a** Safe zone of directional antenna, **b** safe zone of omni-directional antenna

4.1 The successful transmission probability of SUs

We first analyze the SU successful transmission probability denoted by $\mathbb{P}_{SU}^{st}$. Conventionally, The successful transmission of an SU requires no active PUs in its transmission region. However, we consider not only the activities of PUs in an SU's transmission region but also the activities of PUs in a *safe zone* (safe zone or guard zone has been proposed and used in previous work[21, 22]). This consideration makes the transmission of SUs more reliable. As shown in Fig. 3, the shade region represents the transmission region of an SU and the region with solid line represents the transmission region of an SU. More specially, Fig. 3(a) and (b) show the different cases of DIR-CRAHNs and OMN-CRAHNs, respectively.

We then consider the transmission probability. In particular, we have the probability of q packets arriving at a PU in unit time is $\mathbb{P}_{PU}^{arr}(q) = \frac{\lambda_{arr}^q}{q!}e^{-\lambda_{arr}}$ since the traffic arrival event at a PU follows a Poisson process with the density λ_{arr} as given in Sect. 3.1. Thus, the probability that there is no traffic arriving at a PU in unit time is $\mathbb{P}_{PU}^{arr}(0) = e^{-\lambda_{arr}}$.

The probability that there are n PUs falling in the safe zone of an SU is $\mathbb{P}_{PU}(n) = \frac{(\mathcal{A}'\lambda_p)^n}{n!}e^{-\mathcal{A}'\lambda_p}$, where $\mathcal{A}'$ is the area of safe zone of SUs in DIR-CRAHNs according to the following equation

$$\mathcal{A}' = \frac{\theta_s {r_d'}^2}{2} = \frac{\theta_s t^2 r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}, \tag{12}$$

where $r_d' = t \cdot r_d$ and t is the safe zone factor that is no less than 1. Note that the Eq. (12) is a general expression of the area of the safe zone since it can represent both cases of DIR-CRAHNs and OMN-CRAHNs. In particular, when r_d' is equal to r_o' and θ_s is equal to 2π, the area $\mathcal{A}'$ becomes $\pi {r_o'}^2$, i.e., the area of an OMN-CRAHN.

We next derive $\mathbb{P}_{SU}^{st}$, which is essentially equal to the probability that there is no traffic arriving at n PUs. Importantly, there are only $\frac{\theta_p}{2\pi}$ of n PUs can interfere with this SU since the direction of antenna in our model follows a uniform distribution in $[0, 2\pi]$ and PUs are distributed according to a homogeneous Poisson process. Therefore, we have the following equation,

$$\begin{aligned}\mathbb{P}_{SU}^{st} &= \sum_{n=0}^{\infty} \mathbb{P}_{PU}(n) \cdot \mathbb{P}_{PU}^{arr}(0)^{n\cdot\frac{\theta_p}{2\pi}} \\ &= \sum_{n=0}^{\infty} \frac{(\mathcal{A}'\lambda_p)^n}{n!} e^{-\mathcal{A}'\lambda_p} \cdot e^{-\frac{\lambda_{arr} n\theta_p}{2\pi}} \\ &= \exp\left\{-\mathcal{A}'\lambda_p\left(1-e^{-\frac{\lambda_{arr}\theta_p}{2\pi}}\right)\right\} \\ &= \exp\left\{-\frac{\theta_s t^2 r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}\lambda_p\left(1-e^{-\frac{\lambda_{arr}\theta_p}{2\pi}}\right)\right\}.\end{aligned} \tag{13}$$

It is shown in Eq. (13) that the SU successful transmission probability $\mathbb{P}_{SU}^{st}$ is dependent on λ_p, λ_{arr}, θ_s and θ_p in a DIR-CRAHN. Besides, $\mathbb{P}_{SU}^{st}$ is decreasing with $\mathcal{A}'$, λ_p, λ_{arr} and θ_p. When $\theta_s = \theta_p = 2\pi$, our analysis becomes the case of an OMN-CRAHN.

4.2 Probability of node isolation

In this paper, we concern with the *probability of node isolation*, denoted by $\mathbb{P}_{\text{iso}}$, as the metric to measure the local network connectivity [5, 51]. The probability of node isolation is defined as the probability that a node can not connect to any other nodes in the network. According to the link condition of both OMN-CRAHNs and DIR-CRAHNs in Sect. 3.4, we give a general formula of $\mathbb{P}_{\text{iso}}$ in CRAHNs as follows,

$$\mathbb{P}_{\text{iso}} = \sum_{n=0}^{\infty} \mathbb{P}_{SU}(n) \cdot \left((1-\mathbb{P}(e_i)) + \mathbb{P}(e_i)\cdot\left(1-\mathbb{P}(e_j|e_i)\right)^n\right), \tag{14}$$

where $\mathbb{P}_{SU}(n)$ is the probability that there are n neighboring SUs falling in the transmission region of an SU_i and $\mathbb{P}(e_i)$ is the probability of event e_i that an SU_i can successfully transmit, and $\mathbb{P}(e_j)$ is the probability of event e_j that an neighboring SU_j can successfully transmit. Specifically, we have $\mathbb{P}_{SU}(n) = \frac{(\mathcal{A}\lambda_s)^n}{n!}e^{-\mathcal{A}\lambda_s}$ and $\mathbb{P}(e_i) = \mathbb{P}_{SU}^{st}$ from Eq. (13), where $\mathcal{A}$ is the transmission region of an SU is equal to $\frac{\theta_s r_d^2}{2}$.

We next analyze the probability $\mathbb{P}(e_j|e_i)$ that the neighbor SU_j can successfully transmit when SU_i transmits. Note that there are two different cases for DIR-CRAHNs and OMN-CRAHNs, which are shown in Fig. 4(a) and (b), respectively.

Case I: DIR-CRAHNs

Figure 4(a) shows the scenario of any two adjacent nodes in a DIR-CRAHN. If SU_i can transmit successfully, there must be no active PUs falling in its transmission region or the PUs shall not point their antennas toward SU_i even if they fall into SU_i's transmission region. The probability that a PU will direct its antenna toward an SU is equal to $\frac{\theta_p}{2\pi}$. Then, the probability that SU_j can successfully transmit when SU_i transmits is

$$\mathbb{P}(e_j|e_i) = \mathbb{P}(e_i) = \mathbb{P}^{st}_{SU}. \tag{15}$$

Based on the above analysis, we substitute Eq. (15) into Eq. (14) and then obtain the following equation,

$$\begin{aligned}\mathbb{P}_{\text{iso}} &= 1 - \exp\left\{-\frac{\theta_s t^2 r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}\lambda_p\left(1-e^{-\frac{\lambda_{arr}\theta_p}{2\pi}}\right)\right\}\\ &= \exp\left\{-\frac{\theta_s t^2 r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}\lambda_p\left(1-e^{-\frac{\lambda_{arr}\theta_p}{2\pi}}\right)\right.\\ &\quad\left.-\frac{\theta_s r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}\lambda_s\cdot e^{-\frac{\theta_s t^2 r_o^2}{2}\left(\frac{2}{1-\cos\frac{\theta_s}{2}}\right)^{\frac{4}{\alpha}}\lambda_p\left(1-e^{-\frac{\lambda_{arr}\theta_p}{2\pi}}\right)}\right\}.\end{aligned} \tag{16}$$

Case II: OMN-CRAHNs

Figure 4(b) shows the scenario of any two adjacent nodes in an OMN-CRAHN. If SU_i can transmit successfully, there must be no active PUs falling in its transmission region. Different from Case I, there must be no active PUs falling in the intersection of the transmission regions of both SU_i and SU_j since an omni-directional antenna can receive signals from all directions. Therefore, the successful transmission of SU_j only requires the condition that no active PUs fall in the shaded region as shown in Fig. 4(b). We then derive the area of the shaded region as follows,

$$S_O(\theta_o, l) = (\pi-\theta_o){r'_o}^2 + l r'_o \sin\frac{\theta_o}{2} = (\pi-\theta_o)t^2 r_o^2 + l t \sin\frac{\theta_o}{2} r_o, \tag{17}$$

where $\theta_o = 2\arccos\frac{l}{2r'_o} = 2\arccos\frac{l}{2tr_o}$, and l is the Euclidean distance between SU_i and SU_j, which is a random variable. We then have the expectation of S_O given by the follow equation,

$$\mathbb{E}[S_O(l)] = \int_0^{r_o} S_O(l)\cdot f(l)dl. \tag{18}$$

We next calculate the probability density function (PDF) of the distance l between SU_i and SU_j as follows,

$$f_l(l) = \frac{\theta_s \cdot l}{\frac{\theta_s \cdot {r_o}^2}{2}} = \frac{2l}{{r_o}^2}. \tag{19}$$

We then have the $\mathbb{E}[S_O(l)]$ as follows,

$$\mathbb{E}[S_O(l)] = \left(\frac{(1+2t^2)\sqrt{4t^2-1}}{4} - 2t^2(t^2-1)\text{arccsc}(2\text{t})\right)r_o^2 \tag{20}$$

With the above analysis, the probability that SU_j can successfully transmit when SU_i transmits is given by

$$\mathbb{P}(e_j|e_i) = e^{-\lambda_p \mathbb{E}[S_o(l)]\left(1-e^{-\lambda_{arr}}\right)}. \tag{21}$$

Finally, based on the above analysis we substitute Eq. (21) into Eq. (14) and then obtain the following equation,

$$\begin{aligned}\mathbb{P}_{\text{iso}} &= 1-\exp\{-\lambda_p \pi t^2 r_o^2\cdot(1-e^{-\lambda_{arr}})\}\\ &+\exp\left\{-\lambda_p \pi t^2 r_o^2\cdot\left(1-e^{-\lambda_{arr}}\right)-\lambda_s \pi r_o^2\cdot e^{-\lambda_p \mathbb{E}[S_O(l)]\cdot\left(1-e^{-\lambda_{arr}}\right)}\right\},\end{aligned} \tag{22}$$

where $\mathbb{E}[S_O(l)]$ is given by Eq. (20).

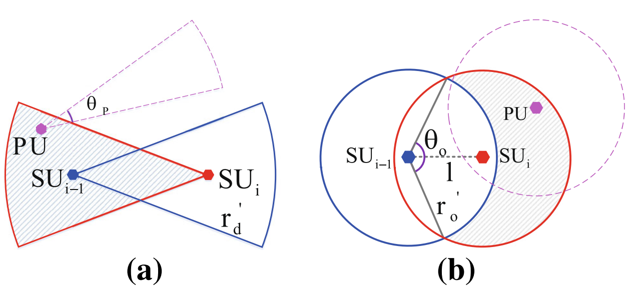

Fig. 4 Local connectivity analysis of two adjacent nodes with different antenna models. **a** Two nodes of DIR-CRAHNs network, **b** two nodes of OMN-CRAHNs network

4.3 Numerical results

In this section, we present several sets of numerical results to evaluate the local connectivity of DIR-CRAHNs in terms of $\mathbb{P}_{\text{iso}}$. We first compare the results between DIR-CRAHNs and OMN-CRAHNs in different density of PUs λ_p, density of SUs λ_s and path loss exponent α, which are shown in Fig. 5. In particular, it is shown in Fig. 5 that the probability of node isolation $\mathbb{P}_{\text{iso}}$ increases with the increased node density of PUs λ_p implying that the connectivity of SNets is decreased. This is mainly due to the increased interference caused by the increased number of PUs. Besides, Fig. 5 also shows that $\mathbb{P}_{\text{iso}}$ decreases with the increased density of SUs λ_s implying that the network connectivity of SNets is improved (Fig. 5a vs. b and c vs. d). Moreover, we can see in Fig. 5 that DIR-CRAHNs

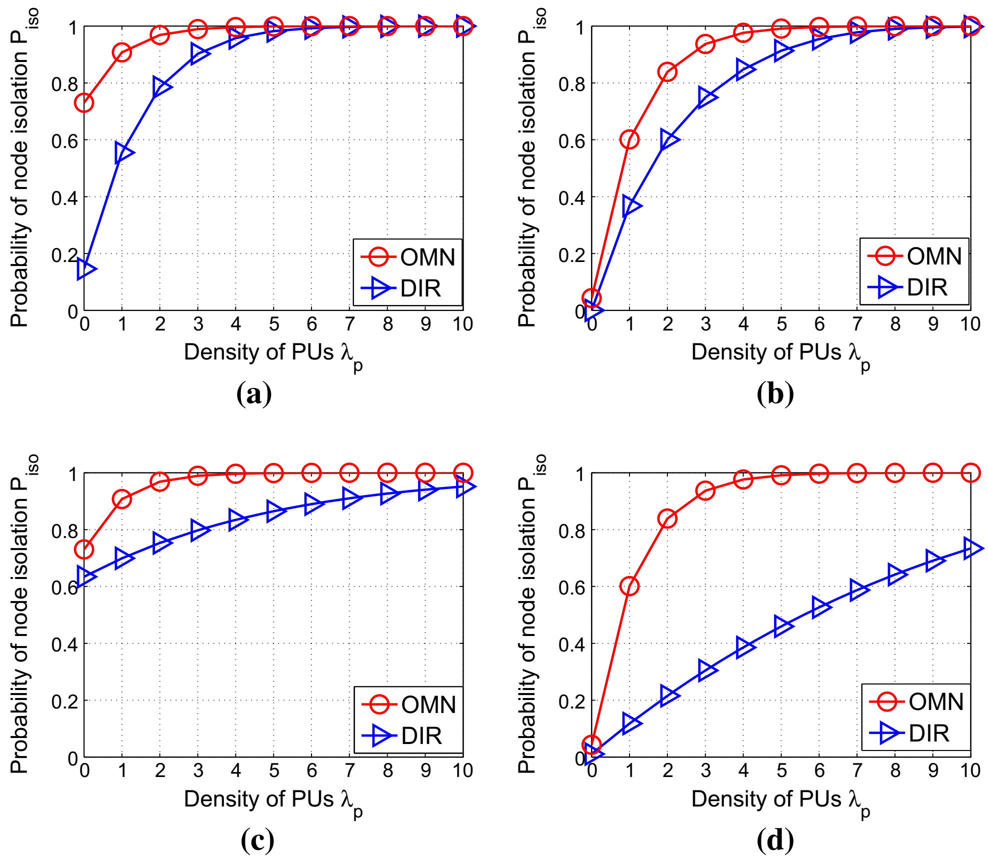

Fig. 5 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs compared with that of OMN-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.2$, $\theta_p = 30°$, $\theta_s = 60°$. **a** $\lambda_s = 0.1$, $\alpha = 3$; **b** $\lambda_s = 1$, $\alpha = 3$; **c** $\lambda_s = 0.1$, $\alpha = 5$; **d** $\lambda_s = 1$, $\alpha = 5$

always have lower probability of node isolation $\mathbb{P}_{\text{iso}}$ than OMN-CRAHNs, indicating that DIR-CRAHNs provide better connectivity than OMN-CRAHNs under different density of PUs λ_p, density of SUs λ_s and path loss exponent α. The improvement of network connectivity of DIR-CRAHNs over OMN-CRAHNs main owes to the reduced interference from PUs to SUs by using directional antennas. We have also obtained another set of numerical results with $\lambda_{arr} = 0.5$ as shown in Fig. 6 (other system parameters are set as the same as that of Fig. 5 for comparison purpose). Figure 5 shows that the probability of node isolation of both OMN-CRAHNs and DIR-CRAHNs is significantly increased when the traffic arrival rate density of PUs is increased (i.e., λ_{arr} is increased from 0.2 to 0.5). This result implies that the activity of PUs has the significant impact on the connectivity of SNets. More specifically, the more active PUs are, the lower connectivity of SNets is.

We then evaluate the network connectivity of DIR-CRAHNs with different system parameters. In particular, Figs. 7 and 8 show the probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs with different path loss exponent values $\alpha = 3$ and $\alpha = 5$, respectively. First, Figs. 7 and 8 show the similar trends to Fig. 5, i.e., $\mathbb{P}_{\text{iso}}$ increases with the increased node density of PUs and decreases with the increased node density of SUs, implying that the network connectivity of SNets heavily depends on PUs' activities. Besides, it is shown in Figs. 7 and 8 that the probability of node isolation $\mathbb{P}_{\text{iso}}$ increases when the antenna beamwidth of PUs θ_p is increased. For example, comparing Fig. 7(a) with Fig. 7(b), we can find that $\mathbb{P}_{\text{iso}}$ increases when the antenna beamwidth θ_p is increased from 30° to 60° while θ_s is fixed. This is because more SUs are suffering from the outage due to the increased interference region of PUs when the antenna beamwidth of PUs θ_p is increased. On the contrary, increasing the antenna beamwidth of SUs θ_s can decrease $\mathbb{P}_{\text{iso}}$, implying the improved network connectivity. Take Fig. 7(a) and (c) as examples again. We find that $\mathbb{P}_{\text{iso}}$ significantly decreases when the antenna beamwidth θ_s is increased from 30° to 60° while θ_p is fixed to be 30°. The network connectivity improvement may owe to the expanded transmission region of SUs, which results in the increased number of neighbors. Furthermore, the local connectivity also depends on the channel condition. For example, increasing the path loss exponent (e.g., α is increased from 3 to 5) can lead to the decrement of $\mathbb{P}_{\text{iso}}$ when we compare

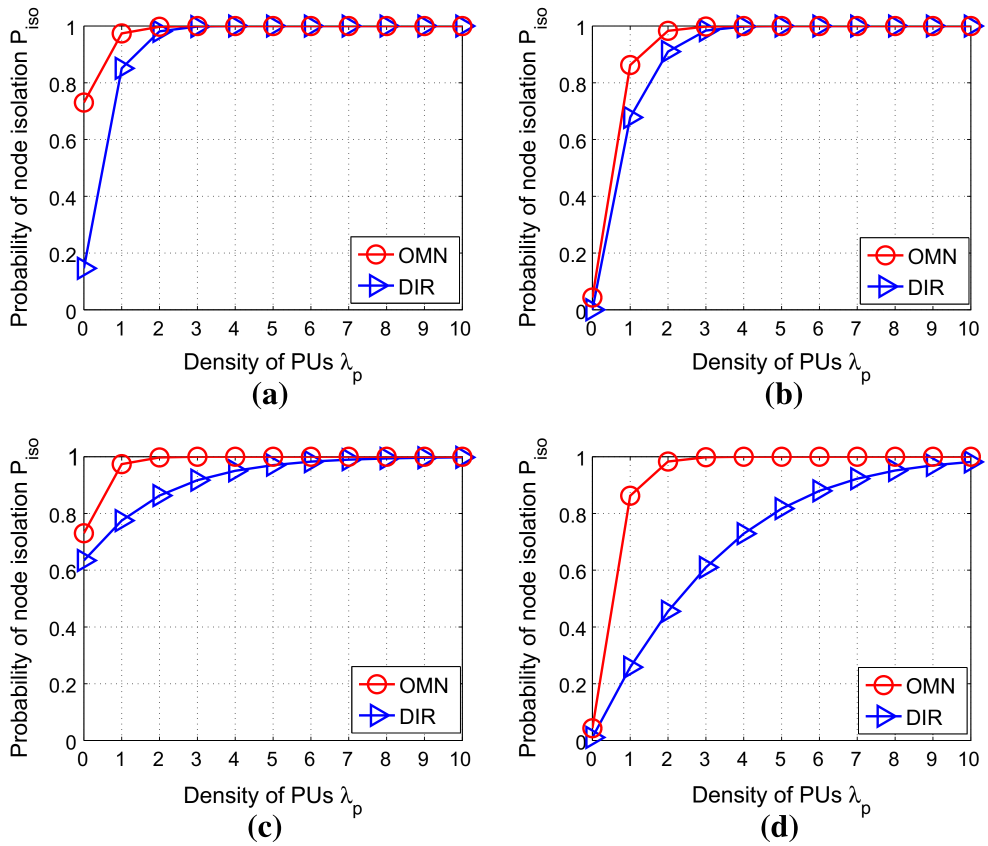

Fig. 6 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs compared with that of OMN-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.5$, $\theta_p = 30°$, $\theta_s = 60°$. **a** $\lambda_s = 0.1$, $\alpha = 3$; **b** $\lambda_s = 1$, $\alpha = 3$; **c** $\lambda_s = 0.1$, $\alpha = 5$; **d** $\lambda_s = 1$, $\alpha = 5$

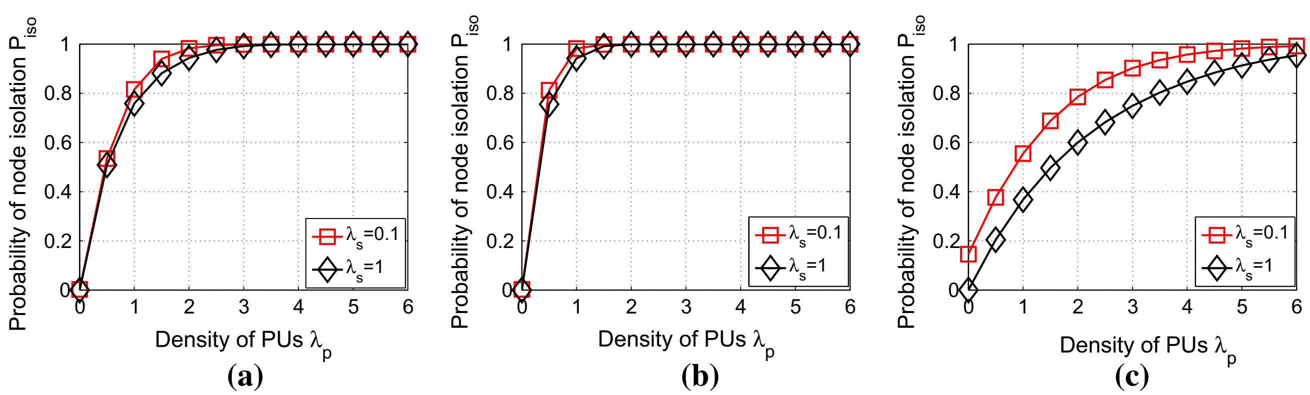

Fig. 7 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.2$, $\alpha = 3$. **a** $\theta_p = 30°$, $\theta_s = 30°$; **b** $\theta_p = 60°$, $\theta_s = 30°$; **c** $\theta_p = 30°$, $\theta_s = 60°$

Fig. 7 with Fig. 8. Similarly, we have also obtained two sets of numerical results with different traffic arrival rate density λ_{arr}. The two sets of results are shown in Figs. 9 and 10 with $\alpha = 3$ and $\alpha = 5$, respectively. If we compare Fig. 9 with Fig. 7 and compare Fig. 10 with Fig. 8, we find that the increased traffic arrival rate density λ_{arr} can significantly increase the probability of node isolation (i.e., the lower connectivity of PNets); this implies that the activity of PUs has the significant impact on the connectivity of PNets.

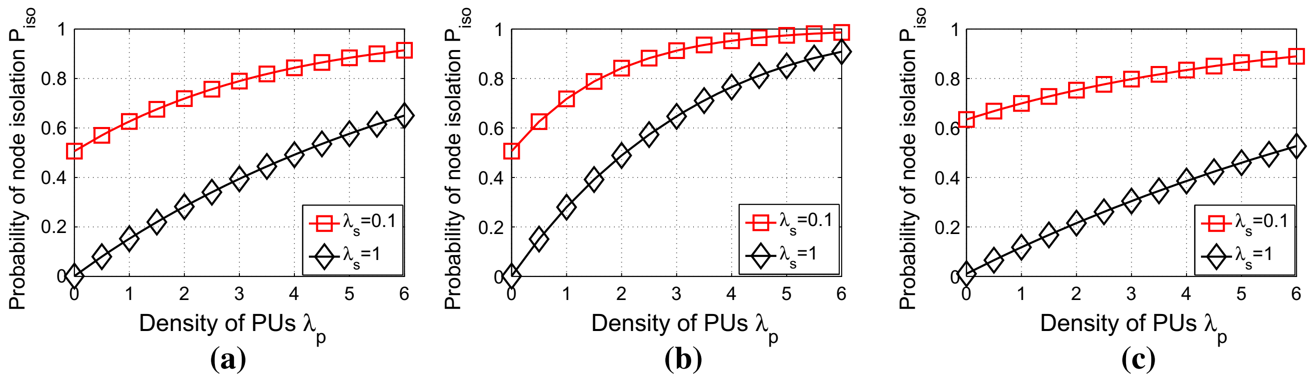

Fig. 8 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.2$, $\alpha = 5$. **a** $\theta_p = 30°$, $\theta_s = 30°$; **b** $\theta_p = 60°$, $\theta_s = 30°$; **c** $\theta_p = 30°$, $\theta_s = 60°$

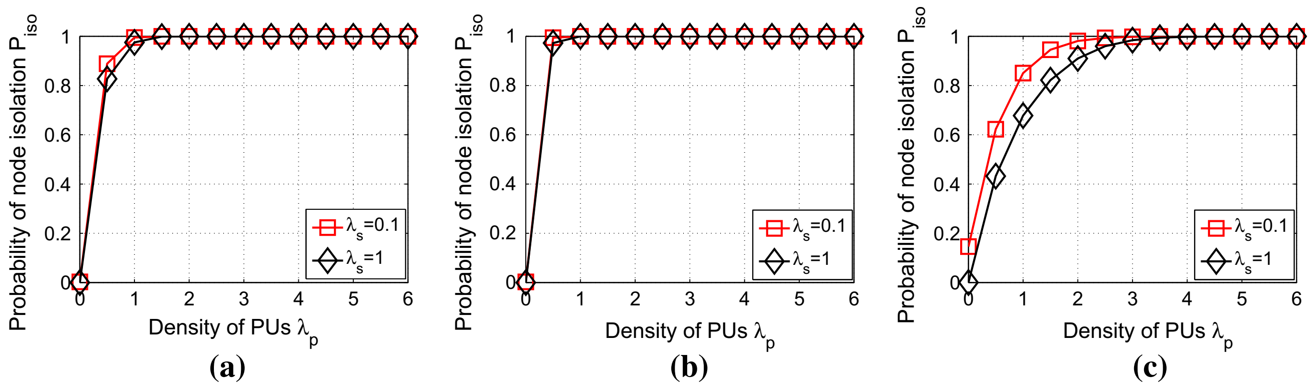

Fig. 9 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.5$, $\alpha = 3$. **a** $\theta_p = 30°$, $\theta_s = 30°$; **b** $\theta_p = 60°$, $\theta_s = 30°$; **c** $\theta_p = 30°$, $\theta_s = 60°$

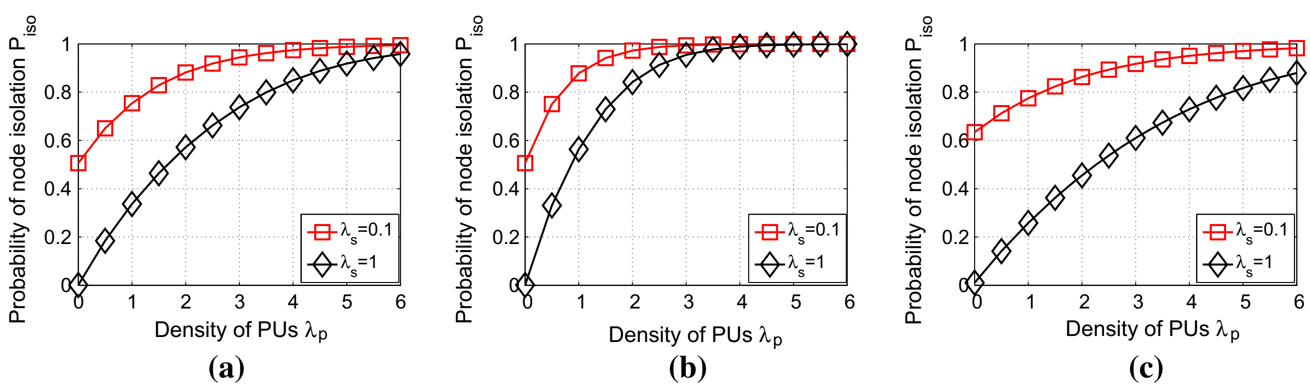

Fig. 10 Probability of node isolation $\mathbb{P}_{\text{iso}}$ of DIR-CRAHNs. System parameters are given as $r_o = 1$, $t = 1.2$, $\lambda_{arr} = 0.5$, $\alpha = 5$. **a** $\theta_p = 30°$, $\theta_s = 30°$; **b** $\theta_p = 60°$, $\theta_s = 30°$; **c** $\theta_p = 30°$, $\theta_s = 60°$

5 Overall network connectivity

In this section, we extend our analysis to the overall network connectivity of both OMN-CRAHNs and DIR-CRAHNs. Section 5.1 presents the analysis of the overall connectivity and Sect. 5.2 gives the analytical results.

5.1 Probability of connectivity

With regard to overall network connectivity, we mainly concern with the probability that there exists a multi-hop path connecting each pair of source node and destination node. It is worth mentioning that each hop consists of a *bidirectional* link as defined in Sect. 3.4 (not a

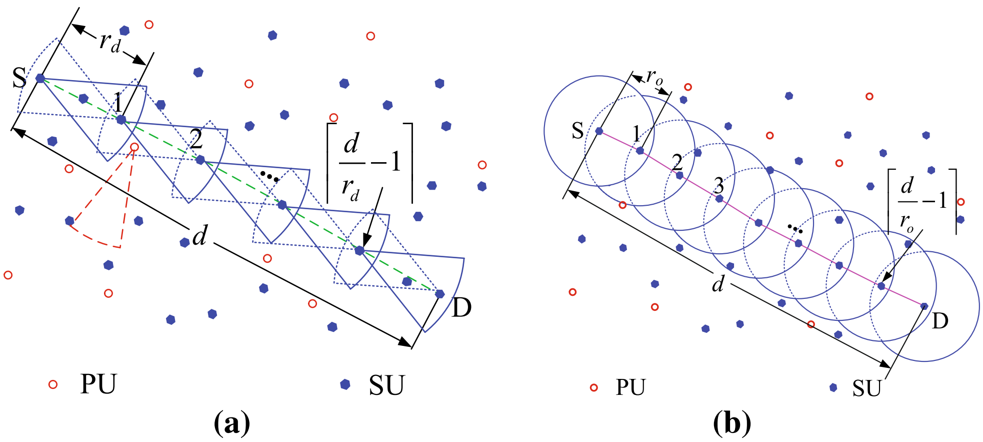

Fig. 11 Routing paths in CRAHNs. **a** Routing path of DIR-CRAHNs, **b** routing path of OMN-CRAHNs

unidirectional link). Take Fig. 11 as an example. We consider the scenario where SU_S communicates with another SU_D that is d distance away from SU_S. To simplify our analysis, we consider that the shortest routing path between SU_S and SU_D can be approximated by a straight line connecting SU_S and SU_D. It is obvious that the length of this straight line is d. We then can bound the minimum number of hops by $\lceil \frac{d}{l} \rceil$, where l is the hop-length. Note that l can be approximated to the transmission range, which is given by the following expression depending on DIR-CRAHNs or OMN-CRAHNs,

$$l = \begin{cases} r_d, & \text{in DIR-CRAHNs} \\ r_o, & \text{in OMN-CRAHNs}. \end{cases}$$

Note that the ith hop only has the correlation with the $(i-1)$th hop, independent of the previous hops. Recall that e_i is the probability of SU_i that can successfully transmit (as defined in Sect. 4.1). We then have a series of events $\{e_S, e_1, \ldots, e_{\lceil \frac{d}{l} \rceil - 1}, e_D\}$ that depict the multi-hop transmission from SU_S to SU_D. Then, the probability of the multi-hop transmission from SU_S to SU_D can be modeled by the successful probability of a series of events $\{e_S, e_1, \ldots, e_{\lceil \frac{d}{l} \rceil - 1}, e_D\}$, which is expressed as follows,

$$\mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_d} \rceil - 1}, e_D) = \mathbb{P}(e_S, e_1)\mathbb{P}(e_1, e_2)\ldots\mathbb{P}(e_{\lceil \frac{d}{r_d} \rceil - 1}, e_D), \tag{23}$$

where $\mathbb{P}(e_i, e_j)$ denotes the probability that SU_i and SU_j can transmit successfully to each other.

We then categorize our analysis into two cases: DIR-CRAHNs and OMN-CRAHNs.

Case I: DIR-CRAHNs

Figure 11(a) shows the routing path of DIR-CRAHNs. According to Eq. (15), we have $\mathbb{P}(e_i, e_j) = \mathbb{P}(e_i)\mathbb{P}(e_j \mid e_i) = {\mathbb{P}_{SU}^{st}}^2$ in DIR-CRAHNs. Then, the probability $\mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_d} \rceil - 1}, e_D)$ can be expressed as

$$\begin{aligned} \mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_d} \rceil - 1}, e_D) &= {\mathbb{P}_{SU}^{st}}^{2 \cdot \lceil \frac{d}{r_d} \rceil} \\ &= e^{-\theta_s t^2 r_o^2 \left(\frac{2}{1-\cos\frac{\theta_s}{2}} \right)^{\frac{4}{\alpha}} \lambda_p \left(1 - e^{-\frac{\lambda_{arr}\theta_p}{2\pi}} \right) \cdot \lceil \frac{d}{r_d} \rceil}, \end{aligned} \tag{24}$$

where $r_d = r_o \left(\frac{2}{1-\cos\frac{\theta_s}{2}} \right)^{\frac{2}{\alpha}}$ (as given by Eq. (11)). Note that Fig. 11(a) shows that there is an overlapping region between the safe zones of two neighbouring nodes, i.e., the overlapping region between two solid sectors (two dash sectors). In this paper, we neglect the overlapping region since this region is so small compared with the transmission region.

Case II: OMN-CRAHNs

Figure 11(b) shows the routing path of OMN-CRAHNs. Note that the hop length l is approximated by r_o. We then have $\mathbb{E}[S_O]$ according to Eq. (17) as the following equation

$$\mathbb{E}[S_O] = S_O = \left(\left(\pi - 2\arccos\left(\frac{1}{2t} \right) \right) t^2 + t \sin\left(\arccos\left(\frac{1}{2t} \right) \right) \right) r_o^2. \tag{25}$$

Fig. 11(b) shows that the safe zone of OMN-CRAHNs is bounded by a circle with radius of r_o' ($r_o' > r_o$). Since the hop length is r_o, we can conclude that if a node (e.g., node 1) in the routing path has the spectrum to communicate with the backward node (e.g., node S), it implies that the node also has spectrum to communicate with the forward node (e.g., node 2). Therefore, we have $\mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_d} \rceil - 1}, e_D) = \mathbb{P}(e_S, e_1)\mathbb{P}(e_1, e_2)\ldots\mathbb{P}$ $(e_{\lceil \frac{d}{r_o} \rceil - 1}, e_D) = P(e_S)P(e_1 \mid e_S)P(e_2 \mid e_1) \ \ldots P(e_D \mid e_{\lceil \frac{d}{r_o} \rceil - 1})$. Then, combining Eqs. (21) and (13), we can obtain the probability of connectivity $\mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_o} \rceil - 1}, e_D)$ as follows

$$\begin{aligned} \mathbb{P}(e_S, e_1, \ldots, e_{\lceil \frac{d}{r_o} \rceil - 1}, e_D) &= \mathbb{P}_{SU}^{st} \cdot e^{-\lambda_p \mathbb{E}[S_o(l)] \left(1 - e^{-\lambda_{arr}} \right) \cdot \lceil \frac{d}{r_o} \rceil} \\ &= e^{-\pi t^2 r_o^2 \lambda_p \left(1 - e^{-\lambda_{arr}} \right) - \mathbb{E}[S_O(l)] \lambda_p \left(1 - e^{-\lambda_{arr}} \right) \cdot \lceil \frac{d}{r_o} \rceil}. \end{aligned} \tag{26}$$

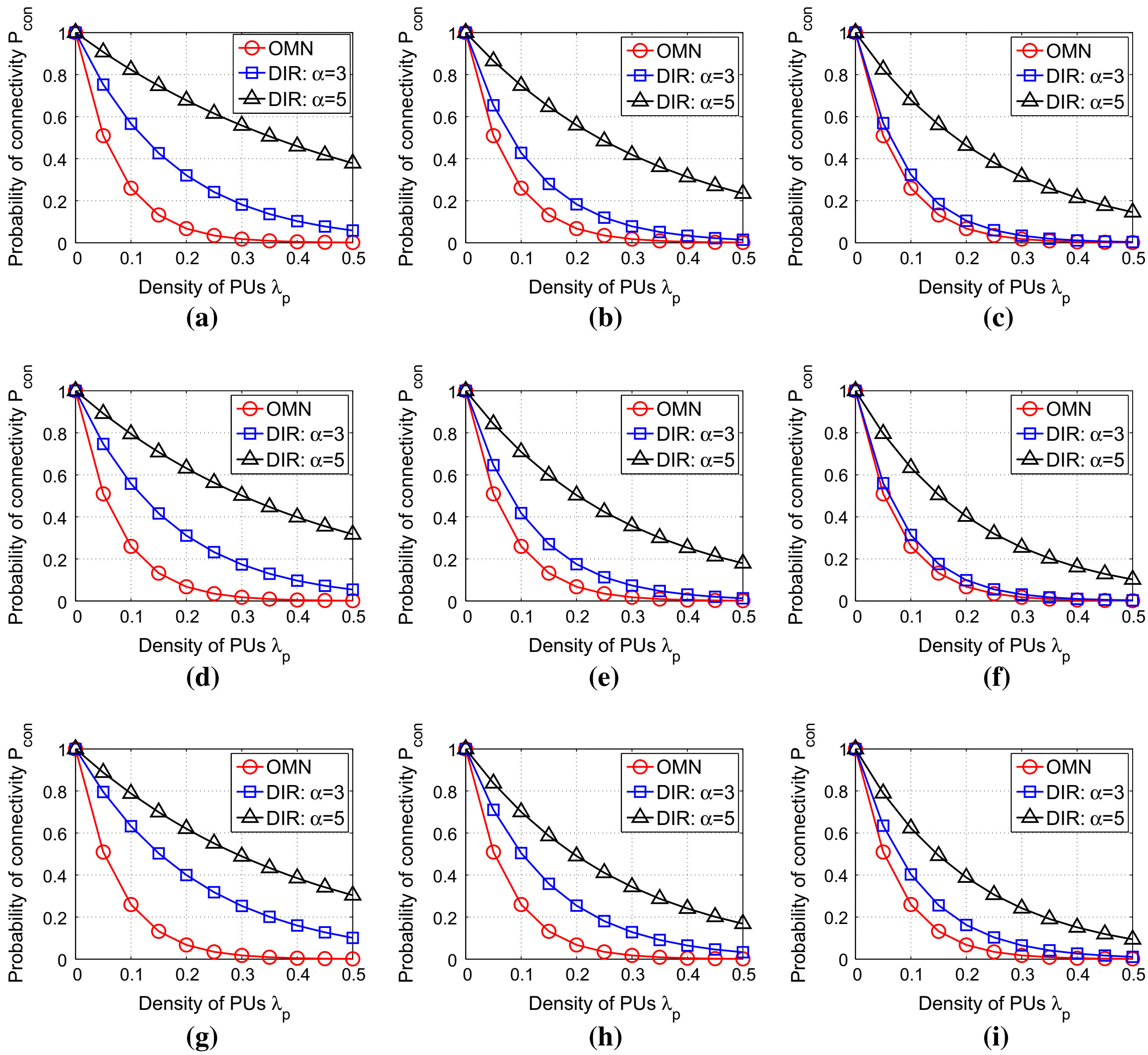

Fig. 12 Probability of connectivity $\mathbb{P}_{\text{con}}$ with DIR-CRAHNs and OMN-CRAHNs. The system parameters are given as $r_o = 1$, $t = 1.2$, $d = 30$, $\lambda_{arr} = 0.2$. **a** $\theta_p = 30°, \theta_s = 30°$; **b** $\theta_p = 45°, \theta_s = 30°$; **c** $\theta_p = 60°, \theta_s = 30°$; **d** $\theta_p = 30°, \theta_s = 45°$; **e** $\theta_p = 45°, \theta_s = 45°$; **f** $\theta_p = 60°, \theta_s = 45°$; **g** $\theta_p = 30°, \theta_s = 60°$; **h** $\theta_p = 45°, \theta_s = 60°$; **i** $\theta_p = 60°, \theta_s = 60°$

5.2 Analytical results

In this section, we present numerical results to compare DIR-CRAHNs with OMN-CRAHNs in terms of the overall connectivity $\mathbb{P}_{\text{con}}$. In particular, Fig. 12 shows the probability of connectivity $\mathbb{P}_{\text{con}}$ versus density of PUs λ_p with the comparison of DIR-CRAHNs and OMN-CRAHNs. It is shown in Fig. 12 that $\mathbb{P}_{\text{con}}$ is decreasing with the increment of PUs' density λ_p due to the interference caused by PUs. These results confirm our previous observations in Sect. 4.3.

Besides, Fig. 12 also shows that DIR-CRAHNs have higher values of $\mathbb{P}_{\text{con}}$ than OMN-CRAHNs in most of cases. Compared with omni-directional antenna, directional antenna can concentrate radio signals on desired directions while there are no radio signals or weakened signals in other undesired directions. As a result, using directional antennas in CRAHNs can significantly improve the network performance due to the reduction on the interference [8, 24, 43, 47].

Furthermore, the network connectivity increases with the increment of the antenna beamwidth θ_p of PUs. For

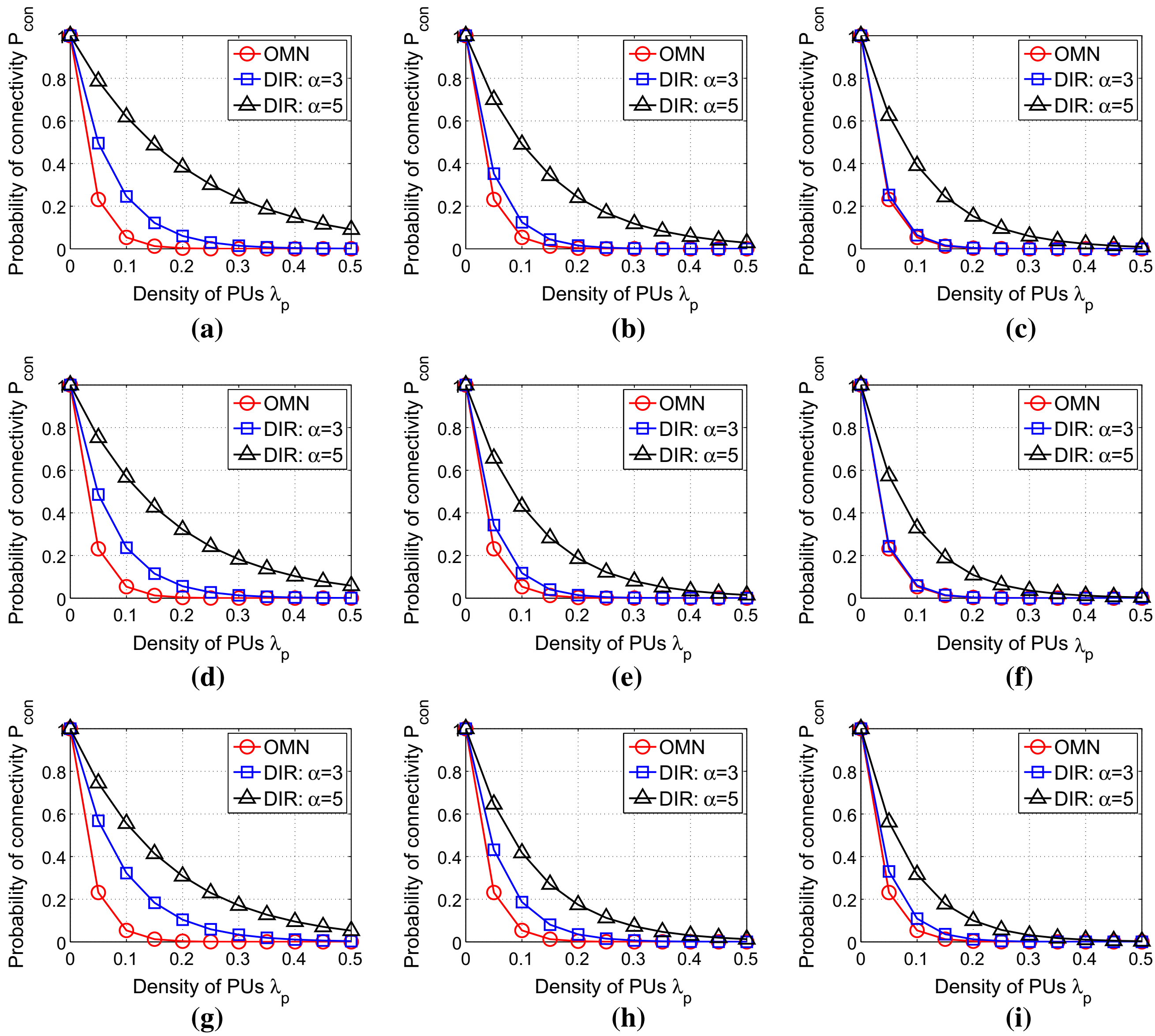

Fig. 13 Probability of connectivity $\mathbb{P}_{con}$ with DIR-CRAHNs and OMN-CRAHNs. The system parameters are given as $r_o = 1$, $t = 1.2$, $d = 30$, $\lambda_{arr} = 0.5$. **a** $\theta_p = 30°, \theta_s = 30°$; **b** $\theta_p = 45°, \theta_s = 30°$; **c** $\theta_p = 60°, \theta_s = 30°$; **d** $\theta_p = 30°, \theta_s = 45°$; **e** $\theta_p = 45°, \theta_s = 45°$; **f** $\theta_p = 60°, \theta_s = 45°$; **g** $\theta_p = 30°, \theta_s = 60°$; **h** $\theta_p = 45°, \theta_s = 60°$; **i** $\theta_p = 60°, \theta_s = 60°$

example, as shown in Fig. 12(a)–(c), with the fixed antenna beamwidth θ_s of SUs (i.e., 30°), $\mathbb{P}_{con}$ significantly increases as the antenna beamwidth θ_p of PUs increases (i.e., increasing from 30° to 60°). This is mainly due to the decreased interference region of PUs with the larger antenna beamwidth θ_p. Besides, increasing the antenna beamwidth θ_s of SUs can slightly increase the overall network connectivity $\mathbb{P}_{con}$ (comparing Fig. 12a, d and g together). This improvement owes to the decreased the numbers of PUs due to the decreased safe zone of SUs when a larger antenna beamwidth θ_s is chosen. Figure 13 shows another set of results with $\lambda_{arr} = 0.5$ (other system parameters are set as the same as that of Fig. 12 for comparison purpose). In contract to Fig. 12, the increased traffic arrival density (i.e., λ_{arr} is increased from 0.2 to 0.5) results in the lower network connectivity of SNets. This implies that the activity of PUs has significant impact on the performance of SNets.

5.3 Implications and future direction

As shown in the empirical results, using directional antennas at PUs can reduce the interference to SUs and consequently improve the spectrum reuse. As a result, the overall connectivity of CRAHNs can be improved. In fact, in mmWave communications (one of possible solutions to 5G communication systems [30, 33, 52], directional transmissions are mandatory to be used since directional antennas can overcome the high attenuation of high-frequency signals. Our results imply that SUs may obtain the higher overall connectivity in mmWave communication systems than those in conventional cellular communication systems (in which omni-directional antennas are used). Therefore, DIR-CRAHNs are one of possible solutions to the spectrum reuse issue of mmWave communication systems.

On the other hand, equipped directional antennas at SUs can also improve the overall connectivity of SUs. This is because directional antennas can extend the transmission region of SUs so that the faraway nodes can be reached by the long links [6]. However, the antenna beamwidth of SUs shall be carefully chosen since too narrow beamwidth can cause the poor connectivity as indicated by our results (e.g., $\theta_s = 30^\circ$ versus $\theta_s = 60^\circ$). One of the future directions is how to choose the proper antenna beamwidth of SUs.

As indicated many recent studies, such as [23, 37], the activity of PUs has significant impacts on the performance of CRAHNs. However, most of current studies on the connectivity of CRAHNs only consider the simplified activity model (e.g., modeling the packet arrival process as a Poisson process) of PUs, which is unrealistic and cannot be used to the real experiments [37]. Therefore, it is one of our future studies to investigate the impact of different activity models of PUs. Furthermore, we are going to investigate the usage of cognitive radio technology on wireless sensor networks (WSNs) in the future since cognitive radio can greatly improve the spectrum reuse in WSNs that is spectrum-ravenous due to the huge number of sensor nodes [18]. In such cognitive radio WSNs, the connectivity is one of the major concerns since the transmission is often conducted in a multi-hop manner. Moreover, we will investigate using the incentive mechanisms in both PUs and SUs so that the performance can be further improved [45, 46].

6 Conclusion

In this paper, we formally establish an analytical framework to investigate both the local connectivity and the overall connectivity of DIR-CRAHNs. Compared with conventional cognitive radio ad hoc network with omni-directional antennas, our DIR-CRAHNs can improve the network connectivity. The connectivity improvement mainly owes to (i) the improved spectrum reuse due to the usage of directional antennas at PUs and (ii) the extended transmission range of SUs by using directional antennas.

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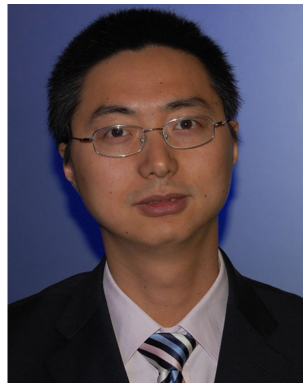

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