On the Connectivity of Wireless Networks with Multiple Directional Antennas

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***Abstract*—The network *connectivity* is one of important measures of the performance of wireless networks. However, most of current studies on the network connectivity only consider either an *SOMN* network, where each node is mounted with a single omni-directional antenna, or an *SDA* network, where each node is mounted with a single directional antenna. Using multiple directional antennas instead of a single directional antenna can potentially improve the network performance. In this paper, we investigate the connectivity of a novel network, in terms of an *MDA* network, where each node is mounted with multiple directional antennas. We found that *MDA* networks have much stronger network connectivity than other existing networks (such as *SOMN* and *SDA* networks), and its connectivity degree heavily depends on the number of antennas, the beamwidth of each antenna and the path loss factor. The enhancement mainly owes to the usage of multiple antennas and the longer transmission range of directional antennas.**

***Keywords*—Wireless Networks; Connectivity; Directional Antennas; Multiple Antennas**

I. Introduction

The network *connectivity* of wireless networks has received a lot of attention recently. The network connectivity is a necessity to ensure the network is connected so that each source node can successfully communicate with its destination node. Besides, the network connectivity is also an important measure of the robustness of a network. However, many studies on the network connectivity [1]–[8] only consider wireless networks consisting of wireless nodes, each of which is mounted with a single omni-directional antenna that can cause high interference. Besides, a single antenna cannot transmit and receive at the same time (i.e., half-duplexity in place). We call such wireless networks using a single omni-directional antenna as *SOMN* networks. An *SOMN* network is suffering from the poor performance, such as the low throughput capacity and the low connectivity.

In contrast to omni-directional antennas, directional antennas can concentrate the radio signal to some directions so that the interference to other undesired directions can be reduced. Many recent works such as [9]–[15] found that applying *directional antennas* instead of omni-directional antennas to wireless networks can greatly improve the network throughput. Besides, some studies [16], [17] found that using directional antennas in wireless networks can significantly improve the network connectivity. However, most of these studies only consider the wireless networks, in which each node is equipped with a single directional antenna (i.e., half-duplexity is still in place). Therefore, the performance improvement of such networks is still limited. We call such wireless networks using a single directional antenna as *SDA* networks.

Using multiple directional antennas at a node in wireless networks can potentially improve the network performance further. For example, [18], [19] found that using multiple directional antennas at each wireless node can significantly improve the network capacity, compared with the existing networks, such as *SOMN* networks and *SDA* networks. We define an *MDA* network to be a wireless network that uses multiple directional antennas and has the following characteristics:

- Each node is equipped with multiple network interface cards (NICs). Each NIC is mounted with a directional antenna.
- All nodes can work in a full-duplex mode, in which a node can simultaneously transmit and receive with different neighbors.

It raises an interesting question: will using multiple directional antennas in wireless networks improve the network connectivity? However, to the best of our knowledge, *there is no study on the network connectivity of MDA networks*. Therefore, the goal of this paper is to investigate the network connectivity of *MDA* networks and to explore the benefits of *MDA* networks. Specifically, our major contributions can be summarized as follows.

1. We are the first to investigate the connectivity of *MDA* networks. In particular, we found that an *MDA* network has a better connectivity than other existing networks, such as *SOMN* networks and *SDA* networks.
2. We found that the network connectivity of such *MDA* networks is mainly affected by the number of nodes, the number of antennas at each node, the beamwidth of each antenna, and the path loss factor.
3. More specifically, we also found that with the increased number of antennas, the network connectivity of an *MDA* network is significantly increased. Besides, the network connectivity of an *MDA* network is also increased when the antenna beamwidth of each antenna is decreased or the number of nodes is increased, but it is decreased with the increased path loss factor.

The remaining paper is organized as follows. Section II gives the models and the problem formulation. Section III presents simulation results. Finally, the paper is concluded in Section IV.

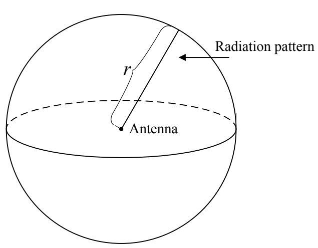


Fig. 1. An omni-directional antenna (an isotropic antenna)

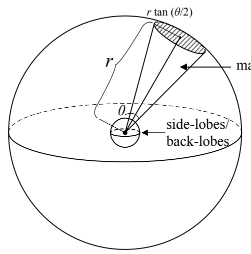


Fig. 2. A directional antenna

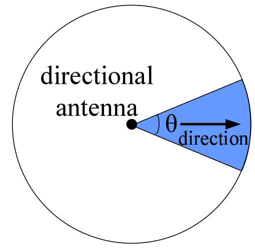


Fig. 3. The Antenna Model

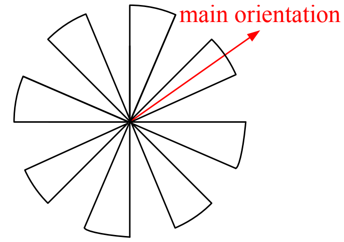


Fig. 4. 8 antennas are uniformly placed at a node without overlaps

II. MODELS

A. Antenna Model

The radiation pattern of a directional antenna is often depicted as the gain value in each direction in space. It typically has a main beam with the peak gain and side/back lobes with smaller gain. Since to model a real antenna with precise values for main beam and side-lobes/back-lobes is difficult, we use an approximate antenna pattern, which was first proposed in [10] and has been widely used in [9], [13], [14], [18], [19]. This approximate antenna model is called cone-sphere model, in which the main beam is model as a cone and the side/back lobes are model as a small bulb at the base of the cone, as shown in Fig. 2.

In order to clarify the analysis on the transmission by using directional antennas, we need to calculate the antenna gain of a directional antennas. The gain value G_m of a main beam is often evaluated by dBi or dB(isotropic), i.e., the antenna gain compared to that of the hypothetical isotropic antenna (denoted by G_i), which uniformly distributes energy in all directions. We assume that both directional antennas and omni-directional antennas are using an identical emanated power P. For an omni-directional antenna (isotropic antenna), as shown in Fig. 1, the transmission power is uniformally emanated in all directions. However, a directional antenna concentrates the energy on a certain direction, i.e., the cone as shown in Fig. 2. Thus, by the definition of the antenna gain, we have

$$\frac{G_m}{G_i} = \frac{\frac{P}{A}}{\frac{P}{S}} = \frac{\frac{P}{\pi r^2 \tan^2 \frac{\theta}{2}}}{\frac{P}{4\pi r^2}} = \frac{4}{\tan^2 \frac{\theta}{2}} \quad (1)$$

where S denotes the surface area of the sphere of the isotropic antenna, r denotes the radius of the sphere, and A denotes the surface area of a directional antenna, which can be approximated as a circle of radius $r \tan \frac{\theta}{2}$ (the gray area in Fig. 2). Given an antenna beamwidth, we can calculate the antenna gains by Eq. (1), where Table I gives the main antenna gains corresponding to the antenna beamwidth. It is shown in Table I that the narrower the beamwidth of a directional antenna is, the higher antenna gain it has.

Similar to other studies [9], [13], [14], [18], [19], we ignore sidelobes and backlobes in this paper. The reasons why we simplify the model are summarized as follows. First, even in a more realistic model, the sidelobes are so small that they can be ignored. For example, the main gain is more than 100 times of the gain of sidelobes when the main beamwidth is less than 40° in the cone-sphere model [10]. Secondly, smart antennas often have null capability that can almost eliminate the sidelobes and backlobes.

TABLE I
THE ANTENNA MAIN GAIN G_m AND THE BEAMWIDTH θ

θ	G_m(dBi)
$\frac{\pi}{3}$	10.797
$\frac{\pi}{4}$	13.681
$\frac{\pi}{6}$	17.464
$\frac{\pi}{8}$	20.047
$\frac{\pi}{10}$	22.026
$\frac{\pi}{12}$	23.637

We then project the antenna pattern in space to an azimuthal plane, where the main lobe of antenna can be depicted as a sector with angle θ, which is denoted as the main beamwidth of the antenna. The gain outside the beamwidth is assumed to be zero. At any time, the antenna beam can only be pointed to a certain direction, as shown in Fig. 3, in which the antenna is pointing to the right. Thus, the probability that the beam is switched to cover each direction is $\theta/2\pi$. More specifically, we have

$$G_d = \begin{cases} G_m & \text{within } \theta \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where G_m can be calculated from Eq. (1). The antenna gain of an omni-directional antenna is equal to that of an isotropic antenna, i.e., $G_o = G_i = 0$ dBi.

In an *MDA* network, each node is equipped with m directional antennas, each of which can independently communicate with other nodes. The antenna model of each antenna can be approximated by the sector model as shown in Fig. 3. Besides, we do not consider the *overlapping* antenna beams at a node, i.e, there is no intersecting area between any two antenna beams of the m antennas. Thus, we have $m \cdot \theta \le 2\pi$. Moreover, to fully utilize the benefits of directional antennas, we uniformly place those m antennas at each node and point the orientation of each antenna centrifugally. Therefore, there is an identical angle between any two adjacent antenna beams, which is equal to $\frac{2\pi - m\cdot\theta}{m}$. For example, as shown in Fig. 4, there are 8 antennas with beamwidth $\theta = \frac{\pi}{8}$ uniformly placed

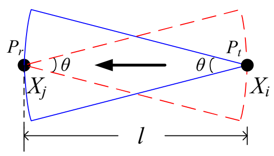


Fig. 5. Link Model

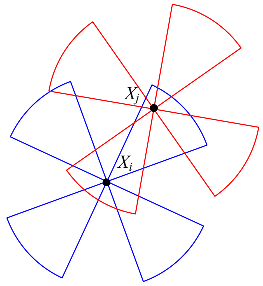


Fig. 6. Link Model of *MDA* networks when $m = 4$

at a wireless node and the intersecting angle between any two adjacent antenna beams is $\frac{\pi}{8}$.

Similar to [16], we also consider *random direction beamforming* in this paper, in which the direction of the antenna beam is randomly chosen from $[0, 2\pi)$. In contrast to [16], we consider random direction beamforming on multiple antennas, called *multiple random beamforming*. In particular, each node arbitrarily chooses one of its m antennas as the main antenna. We then set the orientation of the main antenna as the *main orientation* of a node. The main orientation of a node is randomly chosen from a distribution on the interval $[0, 2\pi)$. Then, the orientations of other $(m - 1)$ antennas can be determined (calculated) by the main orientation since all the m antennas are uniformly placed at a node. For example, as shown in Fig. 4, when the main orientation (the red arrow) of a wireless node is determined and the orientations of the remaining 7 antennas can be easily calculated.

In addition, we also have a uniform setting at each node, i.e., each node in a network is equipped with the same number of antenna, each of which has the same beamwidth θ.

B. Link Model

We then define the wireless link model to determine whether any two given nodes can establish a wireless link. As shown in Fig. 5, we assume that a node X_i transmits with power P_t. The received power is P_r at a node X_j. Therefore, the *path loss*, or the *signal attenuation* [16], [20], in dB is given by

$$PL(\text{dB}) = 10 \log \frac{P_t}{P_r} = 10 \log\left(\frac{1}{G_t \cdot G_r} \cdot \left(\frac{l}{1\text{m}}\right)^{\alpha}\right) \quad (3)$$

where G_t and G_r are the transmitter antenna gain (dBi) and the receiver antenna gain (dBi), respectively, l is the distance between node X_i and node X_j, and α is the path loss factor of the environment, which usually ranges from 2 to 4. Two nodes can establish a link if the path loss PL between them is no greater than the path loss threshold PL_o.

We then define the link model of an *SDA* network, where each node is equipped with a single directional antenna. In this link model, any two nodes X_i and X_j can establish a link if and only if both the following two conditions are satisfied.

(a) The signal attenuation PL between nodes X_i and X_j is no greater than the attenuation threshold PL_0.
(b) The antenna beam of node X_i covers node X_j and the antenna beam of node X_j covers node X_i.

We assume that all nodes have the same threshold PL_0. In this link model, each link is bidirectional, i.e., the receiver X_j can also establish a link to the transmitter X_i if the transmitter X_i can establish a link to the receiver X_j.

Note that this *SDA* link model can be easily extended to an *SOMN* network, where each node is equipped with an omni-directional antenna. In particular, in an *SOMN* network, we will only consider condition (a) and ignore condition (b) since the omni-directional antenna can transmit and receive signals in all directions.

We then extend the *SDA* link model from a node with a single antenna in an *SDA* network to a node with multiple antennas in an *MDA* network. When there are m antennas equipped with each node in an *MDA* network, any two nodes X_i and X_j can form links if and only if all the following conditions are satisfied:

(1) Any one of the m antenna beams of node X_i covers X_j;
(2) Any one of the m antenna beams of node X_j covers X_i;
(2) The signal attenuation PL between node X_i and X_j is no greater than the given attenuation threshold PL_0.

Fig. 6 gives an example of an *MDA* network, where two nodes X_i and X_j can establish a link since one of the four antenna beams of node X_i can cover node X_j, one of four antenna beams of node X_j can cover node X_i and the signal path loss is also less than PL_0.

C. Path Model

We use the *path probability* $P(\text{path})$ to measure the level of network connectivity [16], [21], [22]. The path probability is defined as the probability that two randomly chosen nodes in a random ad hoc network topology can connect each other either through a single-hop link or a multi-hop path. In particular, we can use the following equation to calculate the statistical average of *percentage* of connected node pairs

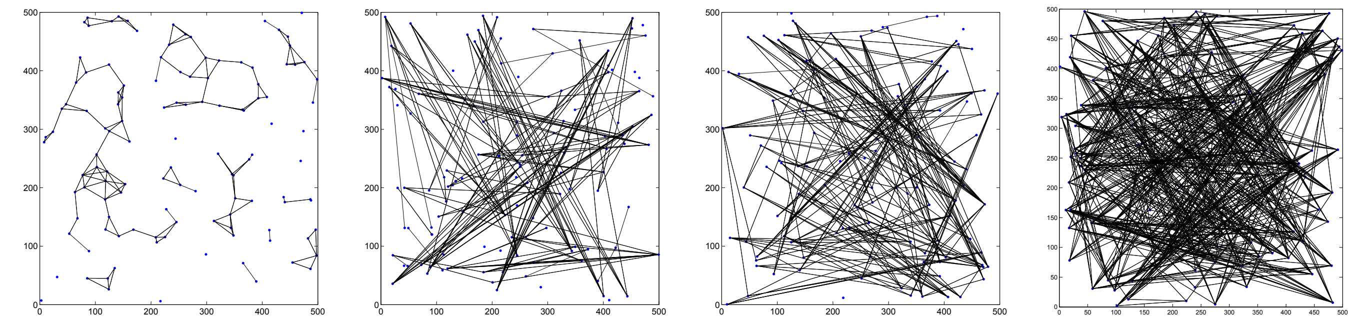
$$P(\text{path}) = \frac{\#\text{ connected node pairs}}{\#\text{ node pairs}} = \frac{\sum_{i=1}^{v} \frac{1}{2} n_i (n_i - 1)}{\frac{1}{2} n (n-1)} \quad (4)$$

where # represents “number of”, n is the total number of nodes in the entire network, v is the number of *connected components*, and n_i is the number of nodes in the ith connected component. After taking the average of the path percentage $P(\text{path})$ over a large number of random topologies, we can obtain an accurate estimation on the path probability. Fig. 7 (a) shows a sample random topology of *SOMN* networks, where there are 11 connected components and several isolated nodes.

It is shown in Eq. (4) that $P(\text{path}) = 1$ if the whole network is completely connected, i.e., there is always a path between any two nodes in the network and therefore $v = 1$ and $n_i = n$. If all the nodes are isolated (i.e., $v = 0$), $P(\text{path}) = 0$. Therefore, the higher path probability $P(\text{path})$ implies the better network connectivity.

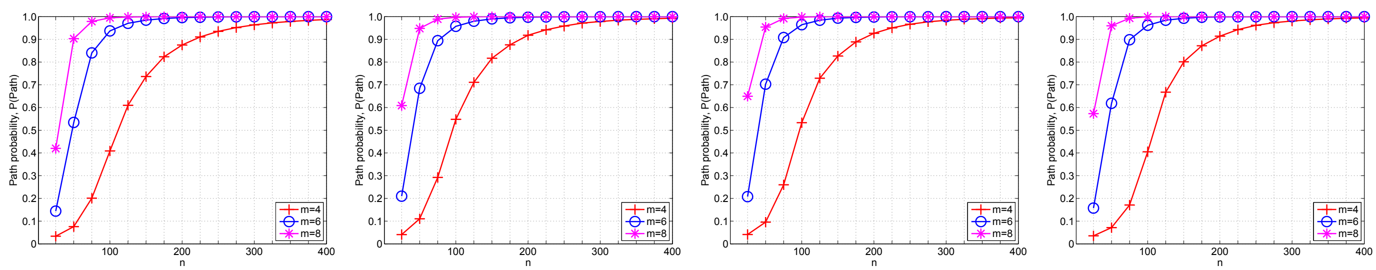
III. Results

In this section, we conduct extensive simulation experiments to investigate the path probability. In particular, Section III-A



(a) An *SOMN* network (b) An *SDA* network ($\theta = \frac{\pi}{6}$ and $m = 1$) (c) An *MDA* network ($\theta = \frac{\pi}{10}$ and $m = 4$) (d) An *MDA* network ($\theta = \frac{\pi}{12}$ and $m = 8$)

Fig. 7. Random topologies with $n = 120$ nodes randomly and uniformly distributed on 500×500 m^2 area when the path loss factor $\alpha = 3$ and the path loss threshold $PL_0 = 50$ dB.



(a) $\theta = \frac{\pi}{6}$ (b) $\theta = \frac{\pi}{8}$ (c) $\theta = \frac{\pi}{10}$ (d) $\theta = \frac{\pi}{12}$

Fig. 8. Path probability $P(\text{Path})$ of n nodes uniformly distributed in an plane with 1000×1000 m^2 area with the path loss factor $\alpha = 3$ and the path loss threshold $PL_0 = 40$ dB. The results are based on 1000 random topologies.

compares the connectivity of *SDA* networks and *MDA* networks. We then study the connectivity of *MDA* networks in Section III-B. Section III-C gives the discussions.

A. Comparison on network connectivity of SDA networks and MDA networks

We place n nodes randomly and uniformly on a plane with area $a \times a$ m^2, where a is the length of this square. We then conduct simulations on *SOMN* networks, *SDA* networks, and *MDA* networks, where we use *random direction beamforming* for *SDA* networks and *multiple random beamforming* for *MDA* networks, as described in Section II-A. Fig. 7 shows example topologies of *SOMN* networks, *SDA* networks, *MDA* networks with $m = 4$, and *MDA* networks with $m = 8$.

It is shown in Fig. 7 (a) that an *SOMN* network contains several connected components and many isolated nodes. Compared with an an *SOMN* network, both an *SDA* network and an *MDA* network contain much larger connected components that span almost all the nodes of the network, as shown in Fig. 7 (b), Fig. 7 (c) and Fig. 7 (d), implying that they have better connectivity than an *SOMN* network. One of possible reasons is that using directional antennas in wireless networks can establish some *long* links to connect the nodes that are far away and even out of the transmission range of an omnidirectional antenna. For example, the average link length of an *SDA* network in Fig. 7 (b) is about 275.6 m, which is much longer than that of an *SOMN* network in Fig. 7 (a), where the average link length of an *SOMN* network is about 67.2 m. This result was also confirmed by [16] although realistic but complicated antenna models are used in [16].

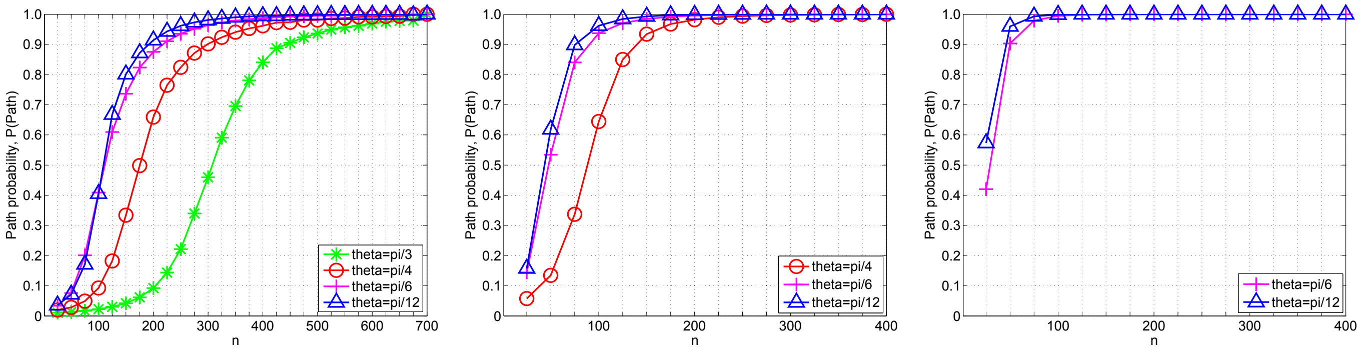
As shown in previous studies [16], [22], a directional antenna may lead to the loss of links to the closely located neighbors, which do not fall in the antenna beam. This implication is also confirmed by our simulations. As shown in Fig. 7 (b), there are a number of nodes, which are isolated but are not far away from the connected components.

Using multiple antennas instead of a single antenna in a network can overcome this negative side effect. For example, there are fewer isolated nodes in an *MDA* network with 4 antennas, as shown in Fig. 7 (c) than an *SDA* network, as shown in Fig. 7 (b). With the increased number of antennas, the network connectivity increases significantly. For example, an *MDA* network with 8 antennas, as shown in Fig. 7 (d) has a higher connectivity than an *MDA* network with 4 antennas, as shown in Fig. 7 (c), where there are almost no isolated nodes in an *MDA* network with 8 antennas.

B. Connectivity of MDA networks

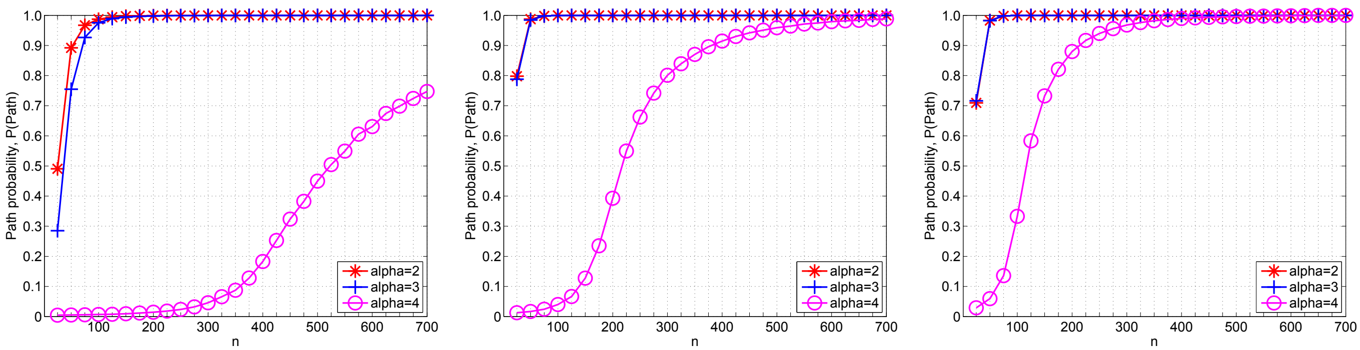
We conduct a number of simulations based on different network settings. We compute each value of the average path percentage $P(\text{Path})$ over 1,000 random topologies, where multiple random beamforming is also considered.

Fig. 8 shows the average path percentage $P(\text{Path})$ against the number of nodes n, the number of antennas m and the



(a) $m = 4$ (b) $m = 6$ (c) $m = 8$

Fig. 9. Path probability $P(\text{Path})$ of n nodes uniformly distributed in an plane with 1000×1000 m^2 area with the path loss factor $\alpha = 3$ and the path loss threshold $PL_0 = 40$ dB. The results are based on 1000 random topologies.



(a) $m = 4$ and $\theta = \frac{\pi}{6}$ (b) $m = 6$ and $\theta = \frac{\pi}{8}$ (c) $m = 8$ and $\theta = \frac{\pi}{12}$

Fig. 10. Path probability $P(\text{Path})$ of n nodes uniformly distributed in an plane with 1000×1000 m^2 area with the path loss threshold $PL_0 = 50$ dB. The results are based on 1000 random topologies.

beamwidth θ. It is shown in Fig. 8 that an *MDA* network with $m = 8$ has a higher network connectivity than an *MDA* network with $m = 6$ and an *MDA* network with $m = 4$ when we choose the beamwidth $\theta = \frac{\pi}{6}$, $\frac{\pi}{8}$, $\frac{\pi}{10}$ and $\frac{\pi}{12}$, respectively. The simulation results further confirm our previous observation that using multiple directional antennas instead of a single directional antenna in the network can significantly improve the network connectivity. This is because multiple directional antennas can avoid the loss of closely located neighbors.

We then investigate what happens if we vary the antenna beamwidth θ when the number of antennas m is fixed. We then conduct the second set of simulations and obtain the results as shown in Fig. 9. Note that the antenna beamwidth θ cannot be "too large" when a larger m is chosen since there is no overlapping between any two adjacent antenna beams. For example, when $m = 8$, we cannot choose beamwidth θ greater than $\frac{\pi}{4}$; otherwise, two antenna beams may overlap each other.

It is shown in Fig. 9 that the path probability $P(\text{Path})$ is significantly increased with the decreased antenna beamwidth θ when other network parameters (e.g., m and α) are fixed. For example, as shown in Fig. 9 (a), an *MDA* network with beamwidth $\theta = \frac{\pi}{12}$ has a higher $P(\text{Path})$ than an *MDA* network with beamwidth $\theta = \frac{\pi}{3}$, an *MDA* network with beamwidth $\theta = \frac{\pi}{4}$ and an *MDA* network with beamwidth $\theta = \frac{\pi}{6}$. This connectivity improvement owes to the long links formed by the narrow beam antennas. As shown in Table I, with the decreased antenna beamwidth, the antenna gain is significantly increased. The increased antenna gain contributes to the signal enhancement over a long distance.

In the third set of simulations, we investigate the path probability when the antenna beamwidth θ and the number of antennas m are fixed and the path loss factor α is varied from 2 to 4. It is shown in Fig. 10 that the network connectivity is also affected by the path loss factor α. With the increased the path loss factor α, the path probability $P(\text{Path})$ is significantly decreased. For example, Fig. 10 (a) shows that when $n = 700$, only 74% of all node pairs are connected when $\alpha = 4$, whereas the path probability $P(\text{Path}) = 100\%$ when $\alpha = 2$ and $\alpha = 3$. This is because when the path loss factor α is large (e.g., an indoor environment), the signal degrades very fast and it is quite difficult to establish links. To overcome the signal loss in such environments, we shall use *more antennas* with *narrower beamwidth*. For example, the connectivity of a network with a larger number of antennas and the narrower beamwidth in Fig. 10 (c) (where $m = 8$ and $\theta = \frac{\pi}{12}$) is higher than that with a smaller number of antennas and the narrower beamwidth in Fig. 10 (b) (where $m = 6$ and $\theta = \frac{\pi}{8}$).

C. Discussions

Our simulations results imply that using narrow beamwidth θ will increase the antenna gain and consequently extend the transmission range so that some remote nodes can be connected. But, on the other hand, the narrow beam antenna can also lead to the loss of nearby neighbors (i.e., fewer neighbors fall into the sector area of an antenna when the beamwidth is narrower). The negative effects may cancel out the benefits if we do not choose the beamwidth properly. For example, we found that the path probability $P(\text{Path})$ with $\theta = \frac{\pi}{12}$ is even lower than that with $\theta = \frac{\pi}{10}$ when $m = 4$ and $n = 100$ nodes randomly distributed on a plane with 1000×1000 m^2. At that time, due to the *border effect*, the transmission range of a node may be out of the border of the plane. The narrower the antenna beamwidth is, the fewer nodes fall into its antenna beam, resulting in the degraded connectivity. One of future issues is to choose a proper beamwidth θ in a practical network.

Our results also show that using more antennas can increase the network connectivity. However, the increased number of antennas may also increase the interference among nodes when it enhances the network connectivity. To avoid the interference, we need to use other technologies to avoid the interference. One of the suggested solutions to reduce the inference in *MDA* networks is to use multiple channels so that the transmissions interfered with each other can be separated in the frequency domain [18]. However, *to the best of our knowledge, there is no practical channel assignment algorithms proposed in MDA networks* though some recent studies, such as [23] investigated the theoretical upper bounds on the number of channels of an *MDA* network. One of future studies is to design a practical channel assignment scheme in *MDA* networks.

Moreover, in this paper, we consider the path model where any two nodes are regarded to be connected only when they are in the same connected component. We do not consider how to build up the routing path between any two nodes to fulfill certain requirements, e.g., the shortest end-to-end delay and the maximum end-to-end throughput. Properly designed routing protocols to fulfill those requirements for *MDA* networks are expected in the future.

IV. Conclusion

In this paper, we investigate the network connectivity of wireless networks using multiple directional antennas (*MDA* networks). We found that the network connectivity of an *MDA* network, in terms of path probability, heavily depends the number of nodes of the network, the number of antennas equipped with each node, the beamwidth of each antenna and the path loss factor. In particular, we found that the network connectivity of *MDA* networks is significantly increased with the increased number of antennas, the decreased antenna beamwidth, the decreased path loss factor. Besides, we also offer some useful suggestions and implications. For example, to enhance the network connectivity, we shall use *more antennas* with *narrower beamwidth* at each node.

Acknowledgement

The work described in this paper was supported by Macao Science and Technology Development Fund under Grant No. 037/2010/A and Grant No. 036/2011/A. The authors would like to thank Guodong Han for his excellent comments.

References

[1] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming*, pp. 547 – 566, 1998.

[2] O. Dousse, P. Thiran, and M. Hasler, "Connectivity in ad-hoc and hybrid networks," in *Proceedings of IEEE INFOCOM*, 2002.

[3] O. Dousse, F. Baccelli, and P. Thiran, "Impact of interferences on connectivity in ad-hoc networks," in *Proceedings of IEEE INFOCOM*, 2003.

[4] O. Dousse and P. Thiran, "Connectivity vs capacity in dense ad-hoc networks," in *Proceedings of IEEE INFOCOM*, 2004.

[5] T. Moscibroda and R. Wattenhofer, "The Complexity of Connectivity in Wireless Networks," in *Proceedings of IEEE INFOCOM*, 2006.

[6] L. Wang, B. Liu, D. Goeckel, D. Towsley, and C. Westphal, "Connectivity in cooperative wireless ad hoc networks," in *Proceedings of ACM MobiHoc*, 2008.

[7] S. Song, D. L. Goeckel, and D. Towsley, "Collaboration Improves the Connectivity of Wireless Networks," in *Proceedings of IEEE INFOCOM*, 2009.

[8] D. Goeckel, B. Liu, D. Towsley, L. Wang, and C. Westphal, "Asymptotic connectivity properties of cooperative wireless ad hoc networks," *IEEE JSAC*, vol. 27, pp. 1226–1237, 2009.

[9] S. Yi, Y. Pei, and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," in *Proceedings of ACM MobiHoc*, 2003.

[10] R. Ramanathan, "On the performance of ad hoc networks with beamforming antennas," in *Proceedings of ACM MobiHoc*, 2001.

[11] M. Takai, J. Martin, R. Bagrodia, and A. Ren, "Directional virtual carrier sensing for directional antennas in mobile ad hoc networks," in *Proceedings of ACM MobiHoc*, 2002.

[12] R. R. Choudhury, X. Yang, N. H. Vaidya, and R. Ramanathan, "Using directional antennas for medium access control in ad hoc networks," in *Proceedings of ACM MOBICOM*, 2002.

[13] T. Korakis, G. Jakllari, and L. Tassiulas, "A MAC protocol for full exploitation of directional antennas in ad-hoc wireless networks," in *Proceedings of ACM MobiHoc*, 2003.

[14] Z. Zhang, "Pure directional transmission and reception algorithms in wireless ad hoc networks with directional antennas," in *IEEE ICC*, 2005.

[15] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, "Ad hoc networking with directional antennas: A complete system solution," *IEEE Journal on Selected Areas in Communications (JSAC)*, vol. 23, no. 3, pp. 496–506, 2005.

[16] C. Bettstetter, C. Hartmann, and C. Moser, "How does randomized beamforming improve the connectivity of ad hoc networks," in *Proceedings of IEEE ICC*, 2005.

[17] P. Li, C. Zhang, and Y. Fang, "Asymptotic Connectivity in Wireless Ad Hoc Networks Using Directional Antenna," *IEEE/ACM Transactions on Networking*, vol. 17, no. 4, pp. 1106 – 1117, 2009.

[18] H.-N. Dai, K.-W. Ng, R. C.-W. Wong, and M.-Y. Wu, "On the Capacity of Multi-Channel Wireless Networks Using Directional Antennas," in *Proceedings of IEEE INFOCOM*, 2008.

[19] J. Zhang and X. Jia, "Capacity analysis of wireless mesh networks with omni or directional antennas," in *IEEE INFOCOM (mini-conference)*, 2009.

[20] T. S. Rappaport, *Wireless communications : principles and practice*, 2nd ed. Upper Saddle River, N.J.: Prentice Hall PTR, 2002.

[21] C. Bettstetter, "On the Connectivity of Ad Hoc Networks," *The Computer Journal*, vol. 47, no. 4, pp. 432 – 447, 2004.

[22] X. Zhou, S. Durrani, and H. Jones, "Connectivity Analysis of Wirelss Ad Hoc Networks with Beamforming," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 9, pp. 5247 – 5257, 2009.

[23] H.-N. Dai, K.-W. Ng, and M.-Y. Wu, "Upper Bounds on the Number of Channels to Ensure Collision-free Communications in Multi-Channel Wireless Networks Using Directional Antennas," in *Proceedings of IEEE WCNC*, 2010.