

Throughput and Delay in Wireless Sensor Networks using Directional Antennas

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Abstract—Most of studies only consider that wireless sensor networks are equipped with only omni-directional antennas, which can cause high collisions. It is shown that the per node throughput in such networks is decreased with the increased number of nodes. Thus, the transmission with multiple short-range hops is preferred to reduce the interference. However, other studies show that the transmission delay increases with the increased number of hops.

In this paper, we consider using directional antennas in wireless sensor networks. We have found that using directional antennas not only can increase the throughput capacity but also can decrease the delay by reducing the number of hops. We also construct a time-division multi-access (TDMA) scheme to achieve this. Compared with omni-directional antennas, directional antennas can reduce the interference and lead to the improvement on the network capacity. Furthermore, directional antennas can extend the transmission range, which leads to fewer hops and the lower multi-hop routing delay.

I. INTRODUCTION

Recently, wireless sensor networks (WSNs) have received enormous interests from both industry and academia [1]. WSNs have been used in environmental monitoring, health care, surveillance security, farming etc. Many studies assume that the sensor nodes are deployed in random from an airplane. Those scattered sensor nodes self-organize to form an ad hoc network, in which data packets are transmitted through multi-hops from the source node to the destination node.

Conventional studies in WSNs often assume an omni-directional antenna equipped with each sensor node. It is shown in [2] that in a wireless ad hoc network with n nodes under a random network configuration, each node has a throughput capacity in the order of $\Theta(1/\sqrt{n \log n})$. Even under an optimal arbitrary network configuration where the location of nodes and traffic pattern can be optimally controlled, the network could only offer a per-node throughput of $\Theta(1/\sqrt{n})$. The per-node throughput is decreased when the number of nodes increases. In fact, all the nodes in such network are sharing the *same* medium to transmit. When a node transmits, its neighboring nodes are prohibited from transmitting due to the interference. Therefore, the network throughput is *interference-limited*.

One implication from [2] is that a small transmission range is necessary to limit the interference and consequently leads to high throughput. Most of recent studies in WSNs assume a small transmission range for each sensor node. However, a smaller transmission range means that a packet needs to be transmitted through more hops, which inevitably leads to

higher transmission delay. Ref. [3] shows that the delay due to the multi-hop transmission is increased when the throughput scales. Thus, increasing the transmission radius can reduce the average number of hops and can reduce the transmission delay. However, the increased transmission range will inevitably cause higher interference which leads to the lower throughput. Thus, there is a trade-off between reducing the delay and improving the throughput.

A lot of studies concentrate on optimizing the trade-off of the delay and the capacity. A similar result to [3] is presented in [4]. Toumpis and Goldsmith [5] propose a scheme that can improve the throughput while the delay is bounded by $O(n^d)$ ($0 < d < 1$). However, all those studies assume that each node is equipped with only omni-directional antennas, which can cause higher interference.

Recent studies, such as [6]–[13] have found that applying directional antennas instead of omni-directional antennas to wireless ad hoc networks can greatly improve the network capacity. In particular, it is shown in [6] that using directional antenna in arbitrary networks achieves a capacity gain of $2\pi/\sqrt{\alpha\beta}$ when both transmission and reception are directional, where α and β are transmitter and receiver antenna beamwidths, respectively. Under random networks, the throughput improvement factor is $4\pi^2/(\alpha\beta)$ for directional transmission and directional reception. However, most of these studies only consider throughput improvement by using directional antennas in wireless ad hoc networks. Although some recent studies consider using directional antennas in wireless sensor networks to improve the network performance [14], to the best of our knowledge, *there is no study addressing the delay due to multi-hop transmission in wireless ad hoc sensor networks using directional antennas*.

In this paper, we study the scaling rules of the delay due to the multi-hop transmission by using directional antennas. The primary research contributions of our paper can be summarized as follows.

- We have studied the capacity improvement by considering the range extension with directional antennas.
- We have also analyzed the delay with directional antennas. We have also compared our results with those derived under omni-directional antennas.
- We have found that using directional antennas not only can significantly increase the network capacity but also can reduce the transmission delay.

The rest of the paper is organized as follows. Section II

presents the models and notations which are used in this paper. In Section III, we describe the analytical results of the delay-throughput trade-off by using directional antennas. Section IV compares our derived results with those of omni-directional antennas. In Section V, we conclude our work and present some interesting problems with directional antennas.

II. MODEL

In this section, we present the antenna model, the propagation model and the interference model, which are used for the analysis in Section III. The definitions and the notations are also given in this section.

A. Antenna Model

The radiation pattern of a directional antenna is often depicted as the gain values in each direction in space. It typically has a main beam with the peak gain and side lobes with smaller gain. Since modeling a real antenna with precise values for main beam and side-lobes/back-lobes is difficult, we use an approximate antenna pattern [7]. In an azimuthal plane, the main lobe of antenna can be depicted as a sector with angle θ , which is denoted as the main *beamwidth* of the antenna. The side-lobes/back-lobes are aggregated to a circle, as shown in Fig. 1. The narrower the main beamwidth of the antenna is, the smaller the side-lobes and back-lobes are. Take the above antenna model as the example. The gain of the main beam is more than 100 times of the gain of side-lobes when the main beamwidth is less than 40° [7]. Thus, the side-lobes and back-lobes can be ignored when the main beamwidth is quite narrow. Therefore, we do not count in the effect of side-lobes and back-lobes this paper.

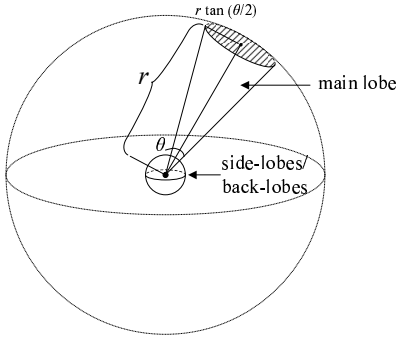


Fig. 1. A directional antenna

In order to clarify the analysis on the transmission by using directional antennas, we need to calculate the antenna gain of a directional antennas. The gain value g_m is often evaluated by dBi or $dB(\text{isotropic})$, i.e., the antenna gain compared to the hypothetical isotropic antenna, which uniformly distributes energy in all directions. First, we derive the antenna gain g_m . We assume that both directional antennas and omni-directional antennas are using an identical emanated power P . For an omni-directional antenna (isotropic antenna), as shown in Fig. 2, the transmission power is uniformly emanated in all directions. However, a directional antenna concentrates the

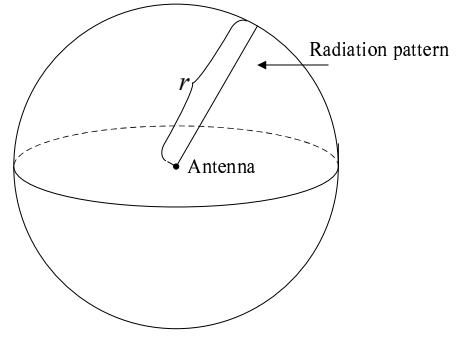


Fig. 2. An omni-directional antenna (an isotropic antenna)

energy on a certain direction, i.e., the cone as shown in Fig. 1. Thus, by the definition of the antenna gain, we have

$$\frac{g_m}{g_i} = \frac{\frac{P}{A}}{\frac{P}{S}} = \frac{\frac{P}{\pi r^2 \tan^2 \frac{\theta}{2}}}{\frac{P}{4\pi r^2}} = \frac{4}{\tan^2 \frac{\theta}{2}} \quad (1)$$

where S denotes the surface area of the sphere of the isotropic antenna, A denotes the surface area of a directional antenna, which can be approximated as a circle of radius $r \tan \frac{\theta}{2}$ (the gray area in Fig. 1). Without loss of generality, the sphere has radius r .

B. Propagation Model

In this paper, we consider that the radio signal follows the large-scale path loss. We propose a general propagation model. The transmitting power is assumed to be P_t . The transmitter and the receiver antenna gains are denoted by G_t and G_r , respectively. The distance between the sender and the receiver is l . Then the signal strength P_r at the receiver is given by

$$P_r = \frac{P_t \cdot G_t \cdot G_r \cdot k_1}{l^\alpha} \quad (2)$$

where k_1 is a constant, and α denotes the path loss factor, which is often ranging from 2 to 4. When $\alpha = 2$, this equation represents the free space propagation model, where there is a clear and unobstructed line-of-sight path between the transmitter and the receiver. When $\alpha = 4$, this equation denotes a two-ray ground propagation model.

To correctly decode the information at the receiver, we require that the received signal strength is greater than a certain value, which is denoted by P_c . Thus, from Eq. (2), we have

$$l_c = \sqrt[\alpha]{\frac{P_t \cdot G_t \cdot G_r \cdot k_1}{P_c}} \quad (3)$$

where l_c is the maximum distance between the transmitter and the receiver when the transmitting power is fixed. From Eq.(3), the maximum distance can be extended when the transmitter or the receiver is equipped with an antenna with higher antenna gain.

Consider the maximum distance when both the transmitter and the receiver are equipped with omni-directional antennas. This distance, denoted as l_{omni} can be calculated by

$$l_{omni} = \sqrt[\alpha]{\frac{P_t \cdot G_i \cdot G_i \cdot k_1}{P_c}} \quad (4)$$

where G_i is the gain of an ideal omni-directional antenna (isotropic) and we assume that both the transmitter and the receiver are equipped with an identical antenna.

Then we calculate the maximum distance when both the transmitter and the receiver are equipped with directional antennas. This distance, denoted as l_{dir} is calculated by

$$l_{dir} = \sqrt[\alpha]{\frac{P_t \cdot G_m \cdot G_m \cdot k_1}{P_c}} \quad (5)$$

where G_m denotes the gain of a directional antenna, which is identical to both the transmitter and the receiver. All the other parameters, such as P_t , P_c , k_1 and α are set to be identical values.

Then we have

$$\frac{l_{dir}}{l_{omni}} = \left(\frac{G_m}{G_i}\right)^{\frac{2}{\alpha}} \quad (6)$$

After replacing $\frac{G_m}{G_i}$ with Eq. (1), we have

$$\frac{l_{dir}}{l_{omni}} = \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{2}{\alpha}} \quad (7)$$

Thus, under the same configuration (the identical transmitting and receiving power and the identical path loss condition), using directional antennas can obtain $\left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{2}{\alpha}}$ range extension, compared with using omni-directional antennas.

C. Receiver-based Interference Model

Based on the protocol model in [2], we propose a receiver-based interference model with extensions of directional antennas. Our model only considers directional transmission and directional reception, which can maximize the benefits of directional antennas.

If node X_i transmits to node X_j , the transmission is successfully completed by node X_j if no nodes within the region covered by X_j 's antenna beam will interfere with X_j 's reception. Therefore, for every other node X_k simultaneously transmitting, and the guard zone $\Delta > 0$, the following condition holds.

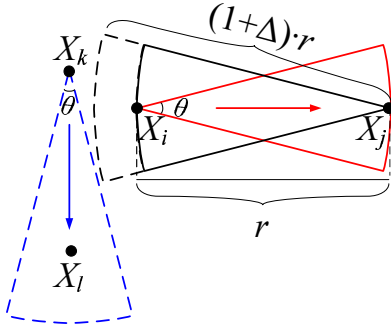


Fig. 3. The Receiver-based Interference Model

$$\begin{cases} |X_k - X_j| \geq (1 + \Delta) |X_i - X_j| \\ \text{or } X_k \text{'s beam does not cover node } X_j \end{cases} \quad (8)$$

where X_i not only denotes the location of a node but refers to the node itself. In this model, each node is equipped with

one single directional antenna. Fig. 3 shows that a transmission from node X_k will not cause interference to X_j 's transmission since the antenna beam of X_k does not cover receiver X_j .

Gupta and Kumar [2] established a physical model in which the success probability of a transmission is related to the Signal-to-Interference-Noise Ratio (SINR). When the *path loss factor* α is greater than two (it is common in a real world), the physical model is equivalent to the interference model. Thus, we will only consider the interference model in this paper.

D. Definitions

In this paper, we adopt the asymptotic notations defined in [15]. Then we define the feasible throughput and the delay due to the multi-hop routing.

Definition 1: Feasible Throughput. A throughput of $\lambda(n)$ bits per second for each node is feasible if every node can send $\lambda(n)$ bits per second on average to its destination. The maximum feasible throughput is $T(n)$ with high probability (*whp*¹).

Definition 2: Delay. The delay of a packet in a network is the time it takes the packet to reach the destination after it leaves the source. In this paper, we just consider the delay due to the routing. Thus, we ignore the queuing delay. We denote $D(n)$ as the average packet delay for a network with n nodes.

In a static network, the delay depends on the sum of the times spent at each relay. In this paper, we ignore the propagation delay since two sensor nodes are quite close in a WSN and the propagation delay between them is quite small. To counteract the dynamics of the network, we take a similar assumption [3], i.e., the service time (transmission delay) is always a constant.

We also adopt the following notations throughout this paper.

- n : the number of nodes.
- θ : the beamwidth of a directional antenna.
- W : the total bandwidth that each node can support.
- λ : each node sends λ bits per second.

III. DELAY AND THROUGHPUT

In this section, we derive the delay and throughput of the networks using directional antennas. We try to analyze the delay-throughput trade-off and compare our results with those derived from networks using omni-directional antennas. First, we construct a time-division multi-access (TDMA) scheme that is similar to those [2] [3]. We then show that our scheme can achieve higher throughput and lower end-to-end delay due to the reduced number of hops.

A. The Delay due to Multi-hop Routing

We consider a random network in which n nodes are randomly placed in a plane of unit area. In such network, each node can randomly choose its destination. First, we divide the unit plane into square cells. Then we design a routing scheme and a transmission scheduling mechanism as follows.

¹In this paper, *whp* means with probability $\geq 1 - 1/n$

- 1) (Torus Division): we divide the unit-area plane into even-sized squares. The size of each square suffices the necessary condition to ensure the network connectivity.
- 2) (Routing Scheme): we construct a simple routing scheme that chooses a route with the shortest distance to forwards packets.
- 3) (Transmission Scheduling): we design a time-division multi-access (TDMA) transmission scheme to ensure a collision-free transmission.

Then we depict these steps in details as follows.

Step 1 (Torus Division): we divide the unit-area plane into a lot of even-sized square cells as shown in Fig. 4. Each of them has an identical area of $a(n)$, which is similar to [3]. The size of the cell, $a(n)$ should be large enough to ensure that there is at least one node in each cell. It is the necessary condition to ensure that the network is connected.

Lemma 1: If $a(n)$ is greater than $2\frac{\log n}{n}$, then each cell contains at least one node *whp*.

This lemma can be proved by using the results in [16]. So, we omit the proof of this lemma in this paper. \square

Then we need to calculate the number of cells that can be affected by a transmission from a cell. We also adopt the definition of *interfering neighbors* introduced by Gupta and Kumar [2].

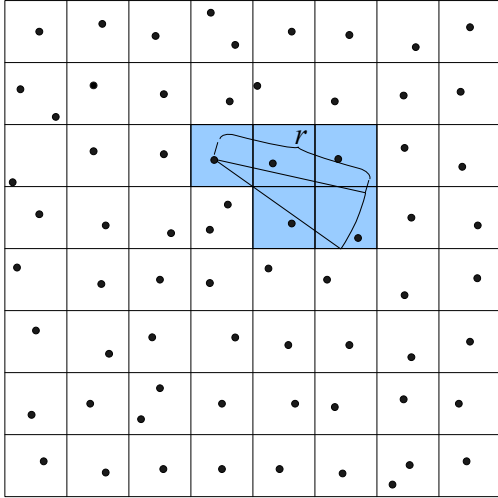


Fig. 4. The interfering neighbors

Definition 3: Two cells are said to be interfering neighbors if there is a point in one cell which is within a distance $(2 + \Delta)r(n)$, where $r(n)$ denotes the transmission range of a node².

This definition implies that if two cells are not *interfering neighbors*, then a transmission from one cell cannot interfere with the transmission from another cell (by *Interference Model*).

As we have mentioned before, using directional antennas can extend the transmission range compared with omni-directional antennas. Intuitively, increasing the transmission

range can cause more cells interfered. However, since a directional antenna can just concentrate the transmission on a certain direction, a directional antenna has a narrower *interfering* angle, compared with an omni-directional antenna. We show that each cell still has a constant number of interfering neighbors even if directional antennas are used.

Lemma 2: Each cell has no more than k_2 interfering neighbors, where k_2 is a constant that depends on θ , Δ and α , but it is independent of n .

Proof. First, we consider a node in a cell transmitting omnidirectionally to another node within the same cell or in one of its eight neighboring cells. Since each cell has area $a(n)$, the distance between the transmitting and receiving nodes cannot be more than $r = \sqrt{8a(n)}$.

From Eq.(7), using a directional antenna can extend the transmission range by $(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}}$. Thus, there should be $(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{4}{\alpha}} \cdot 8a(n)$ nodes that can be affected by this node. However, a directional antenna just concentrates its transmission to a certain direction, as shown in Fig. 4. Thus, only cells covered by the antenna beam of the directional antenna can be interfered (the blue squares in Fig. 4). So, only the proportion of $\frac{\theta}{2\pi}$ of cells can be interfered. Furthermore, since each receiver is also equipped with a directional antenna, it is interfered only when its antenna beam is pointed to the interferer. On average, the probability that a receiver is interfered is $\frac{\theta}{2\pi}$. Thus, there are nearly $(\frac{\theta}{2\pi})^2$ of cells can be interfered.

On the other hand, under the interference model, a packet is successfully received if no node within distance $\bar{r} = (1 + \Delta)r_d$ of the receiver transmits at the same time, where r_d denotes the directional transmission range (in order to discriminate from the omni-directional one). Therefore, the number of interfering cells, k_2 , is at most

$$\begin{aligned}
 k_2 &\leq \left(\frac{\theta}{2\pi}\right)^2 \cdot \frac{\bar{r}^2}{a(n)} \\
 &= \left(\frac{\theta}{2\pi}\right)^2 (1 + \Delta)^2 \cdot \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{4}{\alpha}} \cdot \frac{(r(n))^2}{a(n)} \\
 &= \left(\frac{\theta}{2\pi}\right)^2 (1 + \Delta)^2 \cdot \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{4}{\alpha}} \quad (9)
 \end{aligned}$$

Therefore, there is at most a constant number of cells that can be interfered. \square

Step 2 (Routing Scheme): we construct a simple routing scheme that chooses a route with the shortest distance to forward packets. First, we assign the source and the destination node. For any flow that originates from a cell, source node S is assigned to the flow. Similarly, for any flow that terminates in a cell, destination node D is assigned to the flow. Then we bound the number of such S-D lines passing through a cell.

A source node S sends data packets to its destination D by multi-hop forwarding those packets along the adjacent cells lying on its S-D line. Fig. 5 shows an example of S-D lines, where the green line indicates a transmission from source S to destination D. From this example, we have found that using directional antennas can significantly reduce the

²In random networks, the transmission range is identical to all nodes.

number of hops. For example, only 3 hops is needed from S to D, compared with the omni-directional antenna case, which requires 9 hops from S to D.

Next we derive the bound on the maximum number of S-D lines passing through any cell. The result derived in [3] also holds for the case using directional antennas. Since the proof is presented in [3], we omit the proof here.

Lemma 3: The maximum number of S-D lines passing through any cell is $O(n\sqrt{a(n)})$ whp.

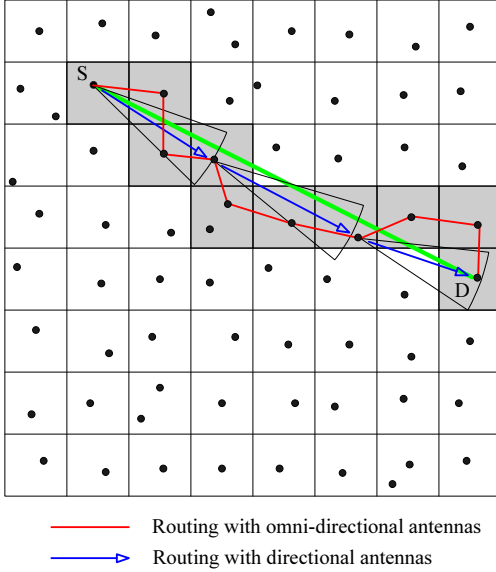


Fig. 5. Using directional antennas can reduce the number of hops

Step 3 (Transmission Scheduling): we propose a TDMA scheme to schedule transmissions. In this scheme, a second is divided into a number of *cell slots* and at most one transmission/reception is scheduled at every cell during each cell slot. Each cell slot can be further split into smaller *mini-slots*. In each mini-slot, a S-D hopping is scheduled.

Fig. 6 depicts a schedule of transmission on the network. Now we describe the process in details.

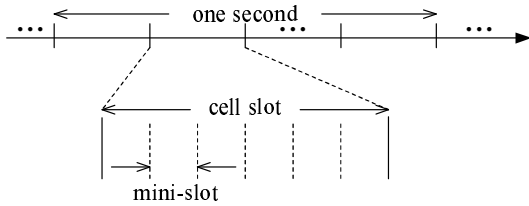


Fig. 6. The TDMA transmission schedule

(i) *Cell slot:* Since the number interfering neighbors of a cell is a constant, each cell can be active for a guaranteed fraction of time. Then, we divide one second into $k_2 = (\frac{\theta}{2\pi})^2(1+\Delta)^2 \cdot (\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{4}{\alpha}}$ cell time slots. Each cell time slot has the length of $\frac{1}{k_2}$.

(ii) *Mini-slot:* Since Lemma 3 suggests that there are at most $O(n\sqrt{a(n)})$ S-D lines passing through one cell, we further

divide each cell slot into $\Theta(n\sqrt{a(n)})$ mini-slots. So, each S-D pair hopping through it can use one mini-slot.

Therefore, each S-D pair can successfully transmit for $\Theta(\frac{1}{k_2 n \sqrt{a(n)}})$ fraction of time. That is, the achieve throughput per S-D pair is $T(n) = \Theta(\frac{1}{k_2 n \sqrt{a(n)}})$.

Then we calculate the average packet delay $D(n)$. As defined before, the packet delay is the sum of the amount of time spent in each hop. First, we derive the bound on the average number of hops.

Since each hop covers a distance of $r_d(n)$, the number of hops per packet for S-D pair i is $\Theta(\frac{d_i}{r_d(n)})$, where d_i is the length of S-D line i and $r_d(n)$ denotes the directional transmission range. Thus, the number of hops taken by a packet averages over all S-D pairs is $\Theta(\frac{1}{n} \sum_{i=1}^n \frac{d_i}{r_d(n)})$. Since for large n , the average distance between S-D pairs is $\frac{1}{n} \sum_{i=1}^n d_i = \Theta(1)$, the average number of hops is $\Theta(\frac{1}{r_d(n)})$. As mentioned before, using directional antennas can extend the transmission range, i.e., $r_d(n) = (\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}} r(n)$, where $r(n)$ is the transmission range by using omni-directional antennas. On the other hand, $r(n)$ is bounded by the edge size of a cell, $\sqrt{a(n)}$. Thus, the average number of hops is bounded by $\Theta(\frac{1}{(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}} \sqrt{a(n)}})$.

From the above discussion, we have the following Theorem.

Theorem 1: For a random network with n nodes equipped with directional antennas, the achievable throughput is $T(n) = \Theta(\frac{1}{k_2 n \sqrt{a(n)}})$ and the average delay is $D(n) = \Theta(\frac{1}{(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}} \sqrt{a(n)}})$.

From Theorem 1, since $a(n)$ needs to be greater than $2 \frac{\log n}{n}$ to ensure the network connectivity, $T(n)$ is still $\Theta(\frac{1}{\sqrt{n \log n}})$.

But there is a capacity improvement factor $\frac{1}{k_2}$, which is brought by directional antennas. Meanwhile, it is shown in Theorem 1 that $D(n) = \frac{k_2 n}{(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}}} T(n)$. The factor $\frac{k_2}{(\frac{4}{\tan^2 \frac{\theta}{2}})^{\frac{2}{\alpha}}}$ is much smaller than 1 since directional antennas have a much narrower beamwidth θ . Therefore, using directional antennas can significantly reduce the delay.

IV. COMPARISONS TO WIRELESS NETWORKS WITH OMNI-DIRECTIONAL ANTENNAS

We also compare our results with those derived under omni-directional antennas. The purpose of this section is to investigate the benefits of using directional antennas in wireless ad hoc sensor networks especially on reducing the delay due to the multi-hop routing.

First, we define the capacity gain factor to quantify the benefits on the throughput capacity by using directional antennas.

Definition 4: Throughput capacity gain factor. The capacity gain factor g_c of a wireless ad hoc sensor network using directional antennas is the ratio of the maximum throughput of such network to that one of a wireless ad hoc sensor network using omni-directional antennas, i.e., $g_c = T_d(n)/T_o(n)$, where $T_d(n)$ represents the achievable throughput of a wireless sensor network consisting of n nodes using directional

antennas, and $T_o(n)$ denotes the throughput of a network with the same number of nodes using omni-directional antennas.

It is shown in [3] that the throughput by using omni-directional antennas is at most $T_o(n) = \frac{1}{n\sqrt{a(n)}}$. Thus, compared with our results, using directional antennas can obtain a capacity gain

$$\begin{aligned} g_c &= T_d(n)/T_o(n) \\ &= \frac{1}{\frac{k_2 n \sqrt{a(n)}}{n\sqrt{a(n)}}} = \frac{1}{k_2} \\ &= \frac{1}{\left(\frac{\theta}{2\pi}\right)^2 (1+\Delta)^2 \cdot \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{4}{\alpha}}} \end{aligned} \quad (10)$$

When the path loss factor $\alpha = 2$, $g_c = \frac{1}{\left(\frac{\theta}{2\pi}\right)^2 (1+\Delta)^2 \cdot \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^2}$. When the antenna beam is quite narrow, i.e., beamwidth θ is quite small, $\tan\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$. Thus, the capacity gain depends on θ^2 , using directional antennas can significantly increase the network throughput. This result is also proved by [6].

When α is larger, e.g., $\alpha = 3$, $g_c = \frac{1}{\left(\frac{\theta}{2\pi}\right)^2 (1+\Delta)^2 \cdot \left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{4}{3}}}$.

The capacity gain by using directional antennas is not so significant. Thus, the improvement on the capacity by using directional antennas is also affected by other factors such as the path loss factor.

Then, we analyze the improvement on the delay due to multi-hop transmissions by using directional antennas. Similarly, we also define the decrease factor on delay.

Definition 5: Delay decrease factor. The delay decrease factor m_d of a wireless ad hoc network using directional antennas is the ratio of the delay of such network to that one of a wireless ad hoc network using omni-directional antennas, i.e., $m_d = D_d(n)/D_o(n)$, where $D_d(n)$ represents the delay of a wireless ad hoc network consisting of n nodes, and $D_o(n)$ denotes that one of a network with the same number of nodes using omni-directional antennas.

Ref. [3] has shown that the delay by using omni-directional antennas is at most $D_o(n) = \frac{1}{\sqrt{a(n)}}$. Compared with our results, using directional antennas can reduce the delay by the factor

$$\begin{aligned} m_d &= D_d(n)/D_o(n) \\ &= \frac{1}{\left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{2}{\alpha}}} \end{aligned} \quad (11)$$

When the path loss factor $\alpha = 2$, the decrease factor is $\frac{1}{\left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{2}{2}}} = \frac{1}{4} \tan^2 \frac{\theta}{2}$. When the beamwidth θ is small, the decrease factor is also quite small, which means that a narrow-beam antenna can reduce the delay further.

When α is larger, e.g., $\alpha = 4$, the decrease factor is $\frac{1}{\left(\frac{4}{\tan^2 \frac{\theta}{2}}\right)^{\frac{2}{4}}} = \frac{1}{2} \tan\left(\frac{\theta}{2}\right)$. Therefore, the narrower antenna beamwidth can still reduce the transmission delay although the decrement is not so significant at this time.

V. CONCLUSION

In this paper, we have studied the throughput and the delay of wireless sensor networks using directional antennas. The goal of this paper is to investigate the benefits by using directional antennas. Our results also apply for the delay analysis on wireless ad hoc network

We have found that using directional antennas not only can improve the network throughput capacity but also can reduce the multi-hop transmission delay. In fact, using directional antennas can significantly reduce the interference, which leads to the throughput improvement. Furthermore, using directional antennas can increase the transmission range, which leads to the reduced number of hops.

There are technical breakthroughs in the miniaturization of directional antennas [17], [18]. Therefore, we may see more applications of directional antennas in wireless ad hoc and sensor networks in the future.

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